# The Relative Advantages of Flexible versus Designated Manufacturing Technologies 

George Norman, Cummings Professor of Entrepreneurship and Business Economics, Tufts University, Medford, MA 02155<br>USA<br>e-mail: george.norman@tufts.edu


#### Abstract

This paper analyzes the choice between flexible and designated manufacturing technologies given that firms are allowed to determine how flexible the manufacturing system should be. We allow firms to operate a mix of technologies, using a flexible system to serve some types of consumer submarkets and a designated technology to serve others and allow firms to offer multiple products even if they commit to the designated technology. We show that for flexible systems to be preferred they must offer strong economies of scope and must be capable of producing, without significant cost penalties, customized products that are largely indistinguishable from custom-built products. By contrast, we show that an increase in submarket size and an increase in the willingness of consumers to pay for particular types of products encourages the use of designated technologies targeted at these submarkets.


(JEL: D4, L1, L2, R1)

## 1. Introduction

Recent years have seen considerable advances in the development and diffusion of flexible manufacturing systems. "Flexibility" in this context refers to flexibility in product design through which manufacturers can adapt a base product to individual consumer requirements at very low additional unit costs: see, for example, Milgrom and Roberts (1990), Mansfield (1993). Specifically, a flexible manufacturing system is defined as:
"a production unit capable of producing a range of discrete products with a minimum of manual intervention" (US Office of Technology Assessment, 1984, p. 60)

Such flexible systems are employed in the manufacture of an increasingly wide range of goods, from ceramic tiles to Levi jeans and custom shoes to data-warehousing. It is now being suggested that such flexibility will find a natural outlet as e-commerce continues to expand. For example, the New York Times recently stated:
"What this means in practice is that rather than displaying the same set of pages to every visitor, a Web site would present different information to each customer based on the person's data profile." (New York Times, 'Internet Companies Learn how to Personalize Services', August 28, 2000)

With the advent of flexible manufacturing systems, technology choice becomes an important strategic issue. The adoption of flexible manufacturing confers advantages that are primarily based upon economies of scope but imposes penalties with respect to the additional setup costs that are necessary to establish such flexible systems: see, for example, Chang (1993), Röller and Tombak (1990, 1993), Norman and Thisse (1999). The existing literature that attempts to address these strategic issues is limited in several respects. In particular, two important questions are not considered.

Given that a firm adopts a flexible manufacturing system
(i) how does it choose the range of products it should offer?
(ii) will a firm wish to operate a mix of flexible and designated technologies?

Question (i) is related to much of the recent literature on product variety in horizontally differentiated industries and leads to another important question:
(iii) with endogenous technology choice will we see product agglomeration as, for example, is discussed in Hamilton, Thisse and Weskamp (1989) and Anderson and Neven (1990)?

This paper attempts to shed some light on these questions. Strategic choice of technology
differentiation. Applications of this approach to flexible manufacturing have been developed by Eaton and Schmitt (1994) and Norman and Thisse (1999). They build on the seminal ideas of
models of product differentiation. MacLeod, Norman and Thisse (1988) show how this analogy has the potential for being applied directly to the strategic analysis of flexible manufacturing:
customers' specifications. This means that the firm now produces a band horizontally differentiated products ... instead of a single product... Transport cost is no longer interpreted as a utility loss, but as an additional cost incurred by the firm in

Almost all of the current literature on flexible manufacturing presents firms with a relatively stark choice. Choose between a flexible or a designated technology. If the designated
an important issue that is central to our analysis. We consider not just the choice between flexible and designated manufacturing technologies but also the choice of just how flexible the
flexible system is capable of producing should be endogenous to the analysis, with firms trading off increased width against the additional set-up costs that increased width imposes. ${ }^{1}$

This leads naturally to a second important element of our analysis. We explicitly allow firms to operate a of technologies, using a flexible system to serve some types of consumer and a designated technology to serve others. Furthermore, we explicitly allow firms to offer
possibility that there will be asymmetry in technology choice in that the strategic choice of flexible systems by one firm will encourage another firm to adopt a different technology choice.
show that the attractiveness of flexible manufacturing is determined by the balance between the economies of scope offered by flexible manufacturing and the economies of scale offered by
designated technologies. Secondly, the advantages of flexible systems are affected by consumer tastes and the ability of flexible manufacturing systems to deliver, at low cost, customized products that are truly substitutable for custom-built products. Thirdly, we show that technology choice is affected by market size and the willingness of consumers to pay for products of particular types.

The remainder of the paper is structured as follows. In the next section we develop the basic model, presenting the choice between designated and flexible technologies as a simultaneous three-stage technology/location/quantity game. Section 3 identifies the subgame perfect Nash equilibria for this game. Finally, we discuss the primary determinants of technology choice in section 4.

## 2. The Model

The demand side is modeled as a variant of the familiar Hotelling (1929) and Salop (1979) analysis in that we assume consumers to be distributed over a line market. But we depart from Hotelling (and the Eaton/Schmitt and Norman/Thisse analyses) in two ways. First, we assume that consumers are concentrated in five evenly spaced submarkets as in Figure $1 .{ }^{2}$ The "distance" between submarkets is designated $r$ (we give a more detailed interpretation of $r$ below). Secondly, demand in each submarket is assumed to be identical and linear: inverse demand in each consumer submarket is:
(1) $\quad p_{i}=a-Q_{i} / s \quad(i=1-5)$
where $s$ is a measure of submarket size.
On the production side we assume that the market is supplied by duopolists who can choose from three technologies differentiated by their "width":
(i) a designated technology (d) with a width of 0 that can be used to serve at most one consumer submarket;
(ii) a partially flexible technology (p) with a width of 1 that can be used to serve a central consumer submarket and at most one submarket on each side of this central submarket;

1 Eaton and Schmitt (1994) do introduce this possibility in the initial specification of their model but drop this cost term in their actual analysis.
2 The reason for our not assuming consumers to be uniformly distributed will become clearer below. The choice of five submarkets is not totally arbitrary. It is large enough to allow consideration of the strategic issues in which we are interested while being small enough to be analytically tractable.
(iii) a flexible technology (f) with a width of 2 that can be used to serve a central consumer submarket and at most two submarkets on each side of this central submarket. ${ }^{3}$

If a firm operates the $\mathbf{d}$ technology to supply some or all of the consumer submarkets the location $i$ of a particular $\mathbf{d}$ product is just the consumer submarket for which the product has been designed. For the $\mathbf{p}$ or $\mathbf{f}$ technologies, $i$ defines the location of the base product on which the $\mathbf{p}$ or $\mathbf{f}$ technology is centered. We use the terminology "designated product $i$ " to refer to the output of ad technology located in consumer submarket $i$ and "base product $i$ " to refer to the consumer submarket on which a $\mathbf{p}$ or $\mathbf{f}$ technology is centered. Consumers in submarket $i$ are assumed to consider a designated product $i$ or base product $i$ produced by either of the duopolists to be perfect substitutes.

Firms can choose to serve some or all consumer submarkets by establishing multiple products. Thus, for example, if technology $\mathbf{d}$ is chosen a firm can choose to establish up to five $\mathbf{d}$ products, if technology $\mathbf{p}$ is chosen the firm can serve the remaining markets by establishing a number of $\mathbf{d}$ products and if technology $\mathbf{f}$ is chosen the firm, if it has not located its base $\mathbf{f}$ product in the central submarket (submarket 3 in Figure 1), can choose to serve the remaining submarkets by establishing additional $\mathbf{d}$ or $\mathbf{p}$ products. When we refer to technology choice $\mathbf{t}$ below we mean the most flexible technology that a firm chooses.
(Figure 1 near here)
In specifying technology costs we distinguish between two types of variable costs. First, there are the variable costs of producing a particular designated or base product - costs of raw materials, intermediate inputs, labor and so on - which can reasonably be assumed to be constant across all three technologies. Without loss of generality these costs are normalized to zero. Secondly there are variable costs of customizing a base product centered in one consumer submarket to the specific requirements of consumers in other submarkets. We consider these costs in more detail below.

In addition to variable costs, each technology is assumed to incur set-up costs. Specifically, a technology of width $w$ is assumed to incur set-up costs of $F(w)$. Clearly:

$$
\begin{equation*}
F(0)<F(1)<F(2) . \tag{2}
\end{equation*}
$$

[^0] containing five submarkets.

We assume that there are economies of scope as the width of the technology increases. In other words,
(3) $\quad F(w)<(2 w+1) \cdot F(0)$ for $w=1,2$.

In the analysis below it will prove convenient to assume that set-up costs increase with width according to a relationship of the form:

$$
\begin{equation*}
F(w)=(1+w \cdot k) \cdot F(0) \quad(i=0,12) \tag{4}
\end{equation*}
$$

where $2 \geq k>0$ is an inverse measure of the economies of scope of the flexible technologies.
We have already indicated that a d product can be sold only in the consumer submarket in which it is located. In the location theoretic interpretation of our model this is equivalent to assuming that the cost of transporting a d product between adjacent consumer submarkets is at least $a$ per unit; in the horizontal product differentiation analogy it is equivalent to assuming that consumers in submarket $j$ consider designated product $i$ to be worth at least $a$ less than designated or base product $j$. By contrast, the cost of transporting a $\mathbf{p}$ or $\mathbf{f}$ product between adjacent submarkets, or equivalently of customizing base product $i$ to the consumer tastes of submarket $(i-1)$ or $(i+1)$ is assumed to be $r$ per unit. ${ }^{4}$

In the product differentiation analogy the parameter $r$ can be thought of as a composite producer and consumer measure of flexibility. From the producer perspective, it seems reasonable to assume that product redesign has some impact on variable costs even where flexible manufacturing systems are introduced. This impact increases the less truly flexible is the flexible technology and the greater the degree of base product redesign that is necessary to customize the base product to the preferences of another consumer submarket (the more differentiated are consumer preferences between submarkets). This element of $r$, in other words, is a combined measure of production flexibility and consumers' preference diversity.

For consumers, $r$ can be considered to contain a measure of the extent to which consumers value customized products that are the output of flexible technologies less than custom-built products, that are the output of designated technologies. ${ }^{5}$ In this latter respect, $r$ is a

[^1]measure of the degree of substitutability between the products of adjacent flexible and designated technologies. Formally we assume:
(5) $r=r_{v}+r_{c}$
where:
$w . r_{v}$ denotes the unit variable costs of redesigning a base product located in submarket $i$ to the desired characteristics of submarket $(i-w)$ or $(i+w)$; and $w \cdot r_{c}$ denotes the extent to which consumers in submarket $(i-w)($ or $(i+w))$ value a product customized from base product $i$ less than a designated or base product located in $(i-w)($ or $(i+w))$.

This implies that if the price of designated (or base) product $i$ is $p(i)$ then the product customized to submarket $i$ from base product $j$ will have to be offered at price $p(i)-r_{c}|i-j|$ and the net revenue received by a firm from the sale of base product $j$ customized to submarket $i$ is $p(i)-\left(r_{v}+r_{c}\right)|i-j|=p(i)-r|i-j|$ per unit.

In characterizing equilibrium we assume that firms 1 and 2 aim to maximize aggregate profit from sales to consumers in the five submarkets through their choices of technology, locations (designs) of their products and outputs. Formally, the technology-location-output game is modeled as a three-stage duopoly game using the concept of subgame perfect Nash equilibrium.

In the first stage firms simultaneously choose their technologies and establish a technology configuration denoted $\mathbf{t}$. In the second stage subgame the firms choose the designs (locations) of their base and/or designated products given the technology configuration established in the first stage. We refer to the outcome of this subgame as a market location configuration, denoted $\mathbf{l}(\mathbf{t})$. The third stage subgame is modeled as a Cournot game in which each firm chooses how much to supply to each consumer submarket (and from which location to supply) given the technology and market location configurations established in the first and second stages. That is, in the third stage subgame we identify the Cournot equilibrium for the two firms in each technology and market location configuration. This timing seems reasonable in that it implies technology choice to be the most inflexible decision and output choice to be the most flexible.

## 3. Equilibrium Technology Choice

We assume that set-up costs are sufficiently low for each firm to be able to supply all consumer submarkets no matter the technology configuration established in the first stage. A sufficient condition for this is $F(0) \leq s . a^{2} / 9$. We further assume that if a firm locates its base $\mathbf{f}$ product at the market center it is able to supply all consumer submarkets no matter the technology choices of its rival, which implies $r \leq a / 4$.

### 3.1 Quantity Equilibrium

Consider any technology and market location configuration (t,l(t)). Firm $m$ supplies consumer submarket $i$ with its product in $(\mathbf{t}, \mathbf{l}(\mathbf{t})$ ) that offers the greatest net revenue per unit. We denote the location of this product $j_{i^{*}}^{m}(\mathbf{t}, \mathbf{l}(\mathbf{t}))$ and the associated quantity supplied to submarket $i$ by $q_{i *}^{m}(i \mid \mathbf{t} \mathbf{l}(\mathbf{t}))$. Standard analysis gives the Cournot-Nash equilibrium quantity supplied to submarket $i$ by firm $m$ as:

$$
\begin{equation*}
q_{i *}^{m *}(i \mid \mathbf{t}, \mathbf{l}(\mathbf{t}))=s\left(a-2\left|i-j_{i *}^{m}\right| r+\left|i-j_{i *}^{n}\right| r\right) / 3 \quad(m, n=1,2, m \neq n) . \tag{6}
\end{equation*}
$$

Price in submarket $i$ is:

$$
\begin{equation*}
p^{*}(i \mid(\mathbf{t}, \mathbf{l}(\mathbf{t})))=\left(a+\left|i-j_{i *}^{m}\right| r+\left|i-j_{i *}^{n}\right| r\right) / 3 \tag{7}
\end{equation*}
$$

and aggregate profit to firm $m$ is:

$$
\begin{equation*}
\Pi_{m}(\mathbf{t})=\sum_{i=1}^{5} \frac{s\left(a-2\left|i-j_{i *}^{m}\right| r+\left|i-j_{i *}^{n}\right| r\right)^{2}}{9}-F_{m}(\mathbf{t}) \quad(m=1,2) \tag{8}
\end{equation*}
$$

where $F_{m}(\mathbf{t})$ is the aggregate set-up costs incurred by firm $m$ in the technology and market location configuration $(\mathbf{t}, \mathbf{l}(\mathbf{t})$ ).

Equation (8) indicates that the subgame perfect equilibrium in our technology-locationoutput game is a function of five parameters $F(0), k, a, s$, and $r$. We can, however, reduce this parameter space to three by formulating the analysis in terms of the demand adjusted parameters:

$$
\begin{equation*}
\rho=r / a ; f()=F() / s \cdot a^{2} \tag{9}
\end{equation*}
$$

in which case aggregate profit in (8) can be rewritten:

$$
\begin{equation*}
\Pi_{m}(\mathbf{t})=s \cdot a^{2}\left(\sum_{i=1}^{5} \frac{\left(1-2\left|i-j_{i *}^{m}\right| \rho+\left|i-j_{i *}^{n}\right| \rho\right)^{2}}{9}-f_{m}(\mathbf{t})\right) \tag{8'}
\end{equation*}
$$

(See Rowthorn (1992) for a similar approach in a different context.) Note that our restrictions on $F(0)$ and $r$ imply that $f(0) \leq 1 / 9$ and $\rho \leq 1 / 4$.

### 3.2 Location Equilibrium

By convention, and without loss of generality, we assume that firm 1 locates to the left of firm 2 if the two firms have not chosen identical locations for their base products. We denote by $\mathbf{L}_{m}(\mathbf{t})$ the set of location configurations that can be chosen by firm $m$ given the technology configuration $\mathbf{t}$. A location configuration $\mathbf{l}_{m}(\mathbf{t}) \in \mathbf{L}_{m}(\mathbf{t})$ is a quintuplet:

$$
\begin{equation*}
\mathbf{1}_{m}(\mathbf{t})=\left(l_{m}^{1}, l_{m}^{2}, l_{m}^{3}, l_{m}^{4}, l_{m}^{5}\right) \quad(m=1,2) \tag{9}
\end{equation*}
$$

where $l_{m}^{i}=d_{m}^{i}, p_{m}^{i}$, or $f_{m}^{i}$ if firm $m$ establishes a designated or base product in submarket $i$ and $l_{m}^{i}=0$ if firm $m$ does not have a designated or base product in submarket $i$.

A market location configuration is a pair $\mathbf{l}(\mathbf{t})=\left\{\mathbf{l}_{1}(\mathbf{t}), \mathbf{l}_{2}(\mathbf{t})\right\}$.
Identification of the Nash equilibrium of the location subgame requires that we consider a number of different possible technology configurations. In doing so we make the further simplifying assumption that the $\mathbf{d}$ technology dominates the $\mathbf{p}$ technology if only two adjacent submarkets are to be supplied and dominates the $\mathbf{f}$ technology if only three adjacent submarkets are to be supplied: a sufficient condition for this to be the case is $k \geq 1 .{ }^{6}$ This assumption has three important implications in our model that considerably simplify the analysis. First, neither firm will locate a $\mathbf{p}$ or $\mathbf{f}$ base product in the most peripheral consumer submarkets 1 and 5 . Secondly, neither firm will combine the $\mathbf{f}$ and $\mathbf{p}$ technologies. Thirdly, neither firm will operate two $\mathbf{p}$ technology plants.

The location subgame for any given technology configuration is analyzed on the assumption that each firm chooses its location(s) to maximize profits given the location(s) of its rival and the equilibrium quantity schedules identified in section 3.1.

### 3.2.1 $t=\{d, d\}$

Given our assumption that $F(0) \leq s a^{2} / 9$, in the technology configuration $\{\mathbf{d}, \mathbf{d}\}$ both firms establish a designated product in every submarket.
$6 \quad$ If $k \geq 1$ then $F(1) \geq 2 . F(0)$ and $F(2) \geq 3 . F(0) . \quad k \geq 1$ is sufficient but not necessary since the $\mathbf{p}$ and $\mathbf{f}$ technologies incur additional costs of $r$ per unit in customizing a base product $i$ to the requirements of adjacent submarkets.

## Lemma 1:

The Nash equilibrium market location configuration for the technology configuration $\{\mathbf{d}, \mathbf{d}\}$ is:

$$
\begin{equation*}
\mathbf{I}_{\mathbf{m}}^{*}(\{\mathbf{d}, \mathbf{d}\})=\left(d_{m}^{1}, d_{m}^{2}, d_{m}^{3}, d_{m}^{4}, d_{m}^{5}\right) \quad(m=1,2) \tag{10}
\end{equation*}
$$

Profit to each firm is:

$$
\begin{equation*}
\Pi_{m}^{*}(\{\mathbf{d}, \mathbf{d}\})=5 s \cdot a^{2}\left(\frac{1}{9}-f(0)\right)(m=1,2) \tag{11}
\end{equation*}
$$

### 3.2.2 $t=\{\mathbf{p}, \mathbf{p}\}$

In analyzing location choice when both firms have chosen the partially flexible technology we can confine our attention to cases in which firm 1 (2) locates its base $\mathbf{p}$ product in consumer submarkets 2 or $3(4$ or 3$) .{ }^{7}$ The pay-off matrix for the location subgame is given in Table 1.

## Lemma 2:

The Nash equilibrium market location configuration for the technology configuration $\{\mathbf{p}, \mathbf{p}\}$ is:

$$
\mathbf{1}_{\mathbf{1}}^{*}(\{\mathbf{p}, \mathbf{p}\})=\left(d_{1}^{1}, 0, p_{1}^{3}, 0, d_{1}^{5}\right) ; \mathbf{i}_{2}^{*}(\{\mathbf{p}, \mathbf{p}\})=\left(d_{2}^{1}, d_{2}^{2}, 0, p_{2}^{4}, 0\right) \text { or }
$$

$$
\begin{equation*}
\mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{p}, \mathbf{p}\})=\left(0, p_{1}^{2}, 0, d_{1}^{4}, d_{1}^{5}\right) \cdot \mathbf{1}_{2}^{*}(\{\mathbf{p}, \mathbf{p}\})=\left(d_{2}^{1}, 0, p_{2}^{3}, 0, d_{2}^{5}\right) \tag{12}
\end{equation*}
$$

with firms 1 and 2 locating their partially flexible base products in consumer submarkets 3 and 4 (or 2 and 3) respectively. ${ }^{8}$

Profit $\Pi_{m}^{*}(\{\mathbf{p}, \mathbf{p}\})$ to each firm is given by the off-diagonal entries in Table 1.
(Table 1 near here)
Lemma 2 indicates that agglomeration, or the principle of minimum differentiation, does not apply when technology is a strategic variable and the technology chosen is not capable, by itself, of supplying the entire market. This is in sharp contrast to cases in which technology is not a choice variable (Hamilton et al. (1989), Anderson and Neven (1990)). The intuition underlying this result can be seen by comparing, for example, the gross profits in each consumer submarket that are made by firm 1 in the location configurations $\left(0, p_{1}^{2}, 0, d_{1}^{4}, d_{1}^{5}\right)$ and $\left(d_{1}^{1}, 0, p_{1}^{3}, 0, d_{1}^{5}\right)$ when firm 2 has chosen the location configuration $\left(d_{2}^{1}, 0, p_{2}^{3}, 0, d_{2}^{5}\right)$. These are given in Table 2.

[^2]Aggregate output is identical for both firms in these two market location configurations (recall equations (6) and (8)) but individual firm profit is determined by the distribution of aggregate output. ${ }^{9}$ No matter the location configuration, firm 1 earns greatest profits from those submarkets in which its product is custom-built rather than customized: submarkets 2, 4 and 5 in the location configuration $\left(0, p_{1}^{2}, 0, d_{1}^{4}, d_{1}^{5}\right)$ and submarkets 1,3 and 5 in the location configuration $\left(d_{1}^{1}, 0, p_{1}^{3}, 0, d_{1}^{5}\right)$. However, in the location configuration $\left(0, p_{1}^{2}, 0, d_{1}^{4}, d_{1}^{5}\right)$ firm 1's custom-built products are in competition with customized products of firm 2 whereas in the location configuration $\left(d_{1}^{1}, 0, p_{1}^{3}, 0, d_{1}^{5}\right)$ they are in competition with custom-built products (with the exception of submarket 5 , of course). The additional profits firm 1 earns in the former location configuration from its custom-built products being in competition with customized products more than offset the lower profits that result from its customized products being in competition with custom-built products. ${ }^{10}$
(Table 2 near here)

### 3.2.3 $t=\{\mathbf{f}, \mathbf{f}\}$

In analyzing location choice when both firms have chosen the wider of the two flexible technologies we can, as with the technology configuration $\{\mathbf{p}, \mathbf{p}\}$, confine our attention to cases in which firm 1 (2) locates its base $\mathbf{f}$ product in consumer submarkets 2 or 3 (4 or 3). The pay-off matrix to the location subgame is given in Table 3.

## Lemma 3:

The Nash equilibrium market location configuration for the technology configuration $\{\mathbf{f}, \mathbf{f}\}$ is:
(i) $f(0) \leq\left(8 \rho-4 \rho^{2}\right) / 9$

$$
\begin{equation*}
\mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{f}, \mathbf{f}\})=\left(0, f_{1}^{2}, 0,0, d_{1}^{5}\right) ; \mathbf{I}_{2}^{*}(\{\mathbf{f}, \mathbf{f}\})=\left(d_{2}^{1}, 0,0, f_{2}^{4}, 0\right) \tag{12}
\end{equation*}
$$

with firms 1 and 2 locating their flexible base products in consumer submarkets 2 and 4 respectively.
(ii) $\left(8 \rho-4 \rho^{2}\right) / 9<f(0) \leq\left(8 \rho+8 \rho^{2}\right) / 9$

[^3]\[

$$
\begin{align*}
& \mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{f}, \mathbf{f}\})=\left(0, f_{1}^{2}, 0,0, d_{1}^{5}\right) ; \mathbf{l}_{2}^{*}(\{\mathbf{f}, \mathbf{f}\})=\left(0,0, f_{2}^{3}, 0,0\right) \text { or } \\
& \mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{f}, \mathbf{f}\})=\left(0,0, f_{1}^{3}, 0,0\right) ; \mathbf{I}_{2}^{*}(\{\mathbf{f}, \mathbf{f}\})=\left(d_{2}^{1}, 0,0, f_{2}^{4}, 0\right) \tag{13}
\end{align*}
$$
\]

with firms 1 and 2 locating their flexible base products in consumer submarkets 2 and 3 (or 3 and 4) respectively. ${ }^{11}$
(iii) $\left(8 \rho+8 \rho^{2}\right) / 9<f(0)$

$$
\begin{equation*}
\mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{f}, \mathbf{f}\})=\left(0,0, f_{1}^{3}, 0,0\right) ; \mathbf{I}_{\mathbf{2}}^{*}(\{\mathbf{f}, \mathbf{f}\})=\left(0,0, f_{2}^{3}, 0,0\right) \tag{14}
\end{equation*}
$$

with firms 1 and 2 each locating their partially flexible base products in consumer submarket 3.

Profit $\Pi_{m}^{*}(\{\mathbf{f}, \mathbf{f}\})$ to each firm is given by the appropriate cell in Table 3.
Agglomeration is the Nash equilibrium to the location subgame with the technology configuration $\{\mathbf{f}, \mathbf{f}\}$ only for $f(0)>\left(8 \rho+8 \rho^{2}\right) / 9$. The reasoning behind this result is straightforward. If the duopolists consider only the variable costs of production then the same forces are at work as those discussed in section 3.2.2, leading to non-agglomeration of the base products. Table 3 indicates that, if set-up costs were ignored, the unique Nash equilibrium market location configuration would be $\left\{\left(0, f_{1}^{2}, 0,0, d_{1}^{5}\right),\left(d_{2}^{1}, 0,0, f_{2}^{4}, 0\right)\right\}$ : recall that $\rho \leq 1 / 4$. Once set-up costs are taken into account, however, there is an additional incentive for either firm to wish to locate at the market center. Central location offers savings in set-up costs. The greater is $f(0)$ the more likely it is that these savings will offset the reduced gross profit that the individual firm makes with more agglomerated locations.
(Table 3 near here)

### 3.2.4 $t=\{d, p\}($ or $\{p, d\})$

Our parameter restrictions imply that the only locations for the $\mathbf{p}$ base product that we need to consider are in submarkets 2,3 or 4 .

## Lemma 4:

The Nash equilibrium market location configuration for the technology configuration $\{\mathbf{d}, \mathbf{p}\}$ is:

[^4]\[

$$
\begin{align*}
& \mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{d}, \mathbf{p}\})=\left(d_{1}^{1}, d_{1}^{2}, d_{1}^{3}, d_{1}^{4}, d_{1}^{5}\right) ; \mathbf{I}_{2}^{*}(\{\mathbf{d}, \mathbf{p}\})=\left(d_{2}^{1}, d_{2}^{2}, 0, p_{2}^{4}, 0\right) \text { or } \\
& \mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{d}, \mathbf{p}\})=\left(d_{1}^{1}, d_{1}^{2}, d_{1}^{3}, d_{1}^{4}, d_{1}^{5}\right) ; \mathbf{I}_{2}^{*}(\{\mathbf{d}, \mathbf{p}\})=\left(d_{2}^{1}, 0, p_{2}^{3}, 0, d_{2}^{5}\right) \text { or }  \tag{15}\\
& \mathbf{I}_{1}^{*}(\{\mathbf{d}, \mathbf{p}\})=\left(d_{1}^{1}, d_{1}^{2}, d_{1}^{3}, d_{1}^{4}, d_{1}^{5}\right) ; \mathbf{I}_{2}^{*}(\{\mathbf{d}, \mathbf{p}\})=\left(0, p_{2}^{2}, 0, d_{2}^{4}, d_{2}^{5}\right)
\end{align*}
$$
\]

with firm 2 indifferent between submarkets 2, 3 and 4 for the location of its partially flexible base product.

Profits in the technology configuration $\{\mathbf{d}, \mathbf{p}\}$ are: ${ }^{12}$

$$
\begin{align*}
& \Pi_{1}^{*}(\{\mathbf{d}, \mathbf{p}\})=\text { s.a } a^{2}\left(\frac{5+4 \rho+2 \rho^{2}}{9}-5 f(0)\right)  \tag{16}\\
& \Pi_{2}^{*}(\{\mathbf{d}, \mathbf{p}\})=\text { s. } a^{2}\left(\frac{5-8 \rho+8 \rho^{2}}{9}-2 f(0)-f(1)\right)
\end{align*}
$$

### 3.2.5 $t=\{d, f\}($ or $\{\mathbf{f}, \mathrm{d}\})$

We need only consider the location of firm 2's $\mathbf{f}$ base product in submarkets 3 or 4. Table 4 gives the profits to the two firms from each location choice by firm 2.

## Lemma 5:

The Nash equilibrium market location configuration for the technology configuration $\{\mathbf{d}, \mathbf{f}\}$ is:
(i) $f(0) \leq\left(8 \rho-16 \rho^{2}\right) / 9$

$$
\begin{equation*}
\mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{d}, \mathbf{f}\})=\left(d_{1}^{1}, d_{1}^{2}, d_{1}^{3}, d_{1}^{4}, d_{1}^{5}\right) ; \mathbf{I}_{2}^{*}(\{\mathbf{d}, \mathbf{f}\})=\left(d_{2}^{1}, 0,0, f_{2}^{4}, 0\right) \tag{17}
\end{equation*}
$$

with firm 2 locating its $\mathbf{f}$ base product in submarket 4;
(ii) $f(0)>\left(8 \rho-16 \rho^{2}\right) / 9$

$$
\begin{equation*}
\mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{d}, \mathbf{f}\})=\left(d_{1}^{1}, d_{1}^{2}, d_{1}^{3}, d_{1}^{4}, d_{1}^{5}\right) ; \mathbf{I}_{2}^{*}(\{\mathbf{d}, \mathbf{f}\})=\left(0,0, f_{2}^{3}, 0,0\right) \tag{18}
\end{equation*}
$$

with firm 2 locating its $\mathbf{f}$ base product in submarket 3.
(Table 4 near here)
The market forces leading to Lemma 5 can be identified from Table 5; they are just those discussed in 3.2.3. If firm 2 chooses the location configuration $\left(0,0, f_{2}^{3}, 0,0\right)$ rather than $\left(d_{2}^{1}, 0,0, f_{2}^{4}, 0\right)$ it gains some additional profit in the central consumer submarkets but loses profit in the peripheral submarkets: in submarket 1 because it has replaced a designated product by a customized product and in submarket 5 because the costs of customizing the base product have
been increased. The resulting lower gross profits of central location are moderated by the lower set-up costs firm 2 incurs and are more than offset if $f(0)>\left(8 \rho-16 \rho^{2}\right) / 9$.
(Table 5 near here)

### 3.2.6 $t=\{p, f\}($ or $\{\mathbf{f}, \mathbf{p}\})$

Our parameter constraints imply that firm 1 will operate a mix of $\mathbf{p}$ and $\mathbf{d}$ technologies, locating its $\mathbf{p}$ base product at 2,3 or 4 , while firm 2 will operate a mix of $\mathbf{f}$ and $\mathbf{d}$ technologies, with the $\mathbf{f}$ base product located at 3 or 4 . The pay-off matrix is given in Table 6.

## Lemma 5:

The Nash equilibrium market location configuration for the technology configuration $\{\mathbf{p}, \mathbf{f}\}$ is:
(i) $f(0) \leq\left(8 \rho-16 \rho^{2}\right) / 9$

$$
\begin{equation*}
\mathbf{I}_{1}^{*}(\{\mathbf{p}, \mathbf{f}\})=\left(0, p_{1}^{2}, 0, d_{1}^{4}, d_{1}^{5}\right) ; \mathbf{I}_{2}^{*}(\{\mathbf{p}, \mathbf{f}\})=\left(d_{2}^{1}, 0,0, f_{2}^{4}, 0\right) \tag{19}
\end{equation*}
$$

with firm 2 locating its $\mathbf{f}$ base product in submarket 4 and firm 1 its $\mathbf{p}$ base product in submarket 2;
(ii) $f(0)>\left(8 \rho-16 \rho^{2}\right) / 9$

$$
\begin{align*}
& \mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{p}, \mathbf{f}\})=\left(0, p_{1}^{2}, 0, d_{1}^{4}, d_{1}^{5}\right) ; \mathbf{l}_{2}^{*}(\{\mathbf{p}, \mathbf{f}\})=\left(0,0, f_{2}^{3}, 0,0\right) \text { or } \\
& \mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{p}, \mathbf{f}\})=\left(d_{1}^{1}, 0, p_{1}^{3}, 0, d_{1}^{5}\right) ; \mathbf{l}_{\mathbf{2}}^{*}(\{\mathbf{p}, \mathbf{f}\})=\left(0,0, f_{2}^{3}, 0,0\right) \text { or }  \tag{20}\\
& \mathbf{I}_{\mathbf{1}}^{*}(\{\mathbf{p}, \mathbf{f}\})=\left(d_{1}^{1}, d_{1}^{2}, 0, p_{1}^{4}, 0\right) ; \mathbf{l}_{2}^{*}(\{\mathbf{p}, \mathbf{f}\})=\left(0,0, f_{2}^{3}, 0,0\right)
\end{align*}
$$

with firm 2 locating its $\mathbf{f}$ base product in submarket 3 and firm 1 indifferent between
locations 2, 3 and 4 for its $\mathbf{p}$ base product.
Once again, agglomeration is a Nash equilibrium only if set-up costs are "sufficiently large".

## (Table 6 near here)

### 3.3 Technology Equilibrium

In the first stage technology choice game each firm chooses its technology to maximize profits given its rival's technology choice and given the equilibrium market location configuration and quantity schedules that will be established in the second- and third-stage subgames.

Lemmas 1-5 indicate that there are five parameter regions to be examined. The technology choice pay-off matrices for each of these parameter regions are given in Appendix Tables A1a-e. Substituting from equation (4) leads to the following:

## Proposition 1:

The three-stage perfect Nash equilibrium is:
(i) $\quad f(0) \leq \frac{8 \rho-8 \rho^{2}}{9(2-k)}$

$$
t^{*}=\{d, d\}
$$

with $\mathbf{I}^{*}\left(\mathbf{t} \mathbf{t}^{*}\right)$ and $q_{i *}^{m *}\left(\mathbf{t} *, \mathbf{1}^{*}(\mathbf{t} *)\right)$ given by (10) and (6) respectively;
(ii) $\frac{8 \rho-8 \rho^{2}}{9(2-k)}<f(0) \leq \frac{16 \rho-24 \rho^{2}}{9(2-k)}$
$t^{*}=\{p, p\}$
with $\mathbf{1}^{*}\left(\mathbf{t}^{*}\right)$ and $q_{i *}^{m *}\left(\mathbf{t}^{*}, \mathbf{1} *(\mathbf{t} *)\right)$ given by (12) and (6) respectively;
(iii) $\frac{16 \rho-24 \rho^{2}}{9(2-k)}<f(0) \leq \frac{16 \rho}{9(2-k)}$
$\mathbf{t}^{*}=\{\mathbf{p}, \mathbf{f}\}$ or $\{\mathbf{f}, \mathbf{p}\}$
with $\mathbf{I}^{*}\left(\mathbf{t}^{*}\right)$ and $q_{i^{*}}^{m^{*}}\left(\mathbf{t}^{*}, \mathbf{1} *(\mathbf{t} *)\right)$ given by (20) and (6) respectively;
(iv) $\frac{16 \rho}{9(2-k)}<f(0)$
$\mathbf{t}^{*}=\{\mathbf{f}, \mathbf{f}\}$
with $\mathbf{I}^{*}\left(\mathbf{t}^{*}\right)$ and $q_{i *}^{m^{*}}\left(\mathbf{t}^{*}, \mathbf{l}^{*}(\mathbf{t} *)\right)$ given by (14) and (6) respectively.
The equilibria of Proposition 1 are illustrated in Figure 2 for the special case of $k=1$.
(Figure 2 near here)

## 4. Discussion of the Determinants of Equilibrium Technology Choice

When competing firms can be multi-product firms no matter their technology choice, the strategic advantages of flexible technologies would appear to be somewhat more limited than previous analysis has suggested. The explanation is straightforward. In our analysis the firms are not forced to adopt flexible manufacturing systems if they wish to supply more than one part

[^5]of the market. Rather, the choice of flexible manufacturing is determined by firms balancing the savings in set-up costs these technologies offer against the variable costs of product customization they impose.

The comparative static effects of changes in market parameters are also affected by our multi-product setting.

From equation (4), the absolute difference in set-up costs between designated and flexible technologies in supplying the same number of consumer submarkets increases with $f(0)$. Proposition 1 and Figure 2 indicate that flexible technologies are more likely to be used the greater is $f(0)$. In other words, and as we would expect, flexible technologies are more likely to be adopted the greater the absolute savings in set-up costs they offer. It is also clear from Proposition 1 and Figure 2 that the boundaries $\mathbf{c}_{\mathbf{n}}(\mathbf{n}=1,2,3)$ fall as $k$ is reduced:

$$
\begin{equation*}
\left.\frac{d f(0)}{d k}\right|_{\mathbf{c}_{\mathrm{n}}}>0 \quad(\mathbf{n}=1,2,3) \tag{21}
\end{equation*}
$$

In other words, the application of flexible manufacturing technologies is encouraged when these technologies offer strong economies of scope relative to the economies of scale offered by designated technologies.

A low value of $\rho$ also leads to the more extensive adoption of flexible manufacturing. This can arise for two reasons (equation (9)): low $r$ or high $a$. Consider the former. (We shall consider below how the demand parameter $a$ affects the technology choice.) Our discussion in section 2 indicates that a low value of $r$ can be attributed to one of three factors. The first is if the variable costs of redesigning a base product $i$ to the desired characteristics of submarket $j$ are low, i.e. if there is a high degree of production flexibility. Secondly, $r$ is low if there is a low degree of differentiation in consumer preferences between consumer submarkets i.e. a high degree of substitutability between adjacent base products. In either of these two cases the producer costs $\left(r_{v}|i-j|\right)$ of customizing base product $i$ to the requirements of submarket $j$ are low. Thirdly, $r$ is lower the greater the extent to which consumers view customized products as being close substitutes for custom-built products i.e. the greater the degree of substitutability between the products of adjacent flexible and designated technologies (recall that this is equivalent to $r_{c}$ in (5) being "low").

This is consistent with the available evidence on the diffusion of flexible manufacturing systems. These appear to be most prevalent in sectors where there is diversity in consumer preferences and where the technology can react fairly accurately to particular consumer requirements. Ceramic tiles, shoes, automobiles and housing are obvious examples. It also suggests that e-commerce should encourage the spread of flexible systems by providing accurate information on consumer tastes while at the same time offering relatively inexpensive technologies that allow service providers to target these tastes accurately.

Now consider the effect of submarket size $s$. Recall that $f(0)=F(0) / s . a^{2}$ so that $f(0)$ is a decreasing function of submarket size. It follows that an increase in submarket size reduces the strategic incentive to adopt flexible manufacturing technologies. This might at first sight appear to be counter-intuitive and is certainly counter to results derived from alternative specifications of flexible technology choice (see, for example, Röller and Tombak (op. cit.)). The explanation lies in the tension in our model between economies of scope and economies of scale. While only the flexible technologies exhibit economies of scope, all three technologies exhibit economies of scale. Increased submarket size allows firms to take greater advantage of economies of scale no matter their technology choice and reduces the demand-adjusted cost disadvantage of the designated technology. In other words, the larger are the individual consumer submarkets the greater the incentive a firm has to design designated, or niche, products for these submarkets rather than supply them with products that are customized versions of a base product targeted at other submarkets. When seen in this light, this outcome seems to accord well with our intuition.

Technology choice is affected in a similar manner by the consumer reservation price $a$. Note from (9) that both $f(0)$ and $\rho$ are declining functions of $a$. In other words, an increase in the consumer reservation price reduces the variable costs of production flexibility but also reduces the set-up cost advantage of production flexibility. The set-up cost effect is greater than the variable cost effect with the result that an increase in the consumer reservation price reduces the strategic incentive to adopt flexible manufacturing techniques. Simply put, a greater (lesser) willingness on the part of consumers to pay for products designed to their specific requirements encourages the adoption of designated (flexible) technologies.

This points to an offsetting influence that will tend to limit the diffusion of flexible manufacturing. As incomes rise consumers can be expected to be willing to pay more for
products designed to their specific tastes, encouraging firms to adopt technologies that are well adapted to satisfying niche markets. This is, presumably, one of the reasons why highly specialized fashion houses such as Dior and Escada are able to thrive in high-income markets while more flexible companies such as Wal-Mart thrive in lower income environments.

## 5. Conclusions

Flexible manufacturing systems capable of customizing products to the tastes of heterogeneous consumers are generally regarded as being superior to technologies that are capable only of producing designated or niche products. If that is so then we should expect over time that flexible technologies will drive out inflexible ones.

There are, however, reasons for questioning this apparently appealing conclusion. In earlier work Norman and Thisse (1999) argue that flexible manufacturing leads to a more competitive pricing regime with the result that adoption of such technologies might actually reduce profitability. Our analysis in this paper suggests other reasons for skepticism regarding the evolutionary dominance of flexible systems. We have argued that the analysis of competing technologies should allow for the possibility that firms offer multiple products, are able to choose the degree of flexibility of any flexible system that they employ, and are able to deploy a mix of technologies.

This leads to a rather more complex set of trade-offs. It remains the case that flexible technologies are preferred when they offer strong economies of scope relative to the economies of scale available from designated technologies and when the customized products of flexible technologies can be made practically indistinguishable from custom-built products at very little cost penalty. On the other hand, we have shown that flexible technologies are not necessarily preferred when consumers differ widely in their preferences. In such circumstances it may well be better for the firm to offer multiple designated products, each targeted at a particular part of the taste spectrum.

By a similar argument, our analysis suggests that as particular parts of the consumer taste spectrum grow in size and as consumer incomes rise it may be better for firms to produce niche products designed specifically for particular submarkets rather than try to serve these markets by using flexible manufacturing to customize a base product centered in another submarket. In other words, we would argue that the likely future scenario is likely to exhibit a high degree of
heterogeneity in the technologies that firms employ, with firms will operate a range of technologies of varying degrees of flexibility determined by the precise characteristics of the markets they are trying to capture and the ability of the flexible technology accurately to target particular consumer requirements.

## References

Anderson, S.P. and D.J. Neven (1990) Cournot competition yields spatial agglomeration, International Economic Review, 32, 793-808
Chang, M.H. (1993) Flexible manufacturing, uncertain consumer tastes, and strategic entry deterrence, The Journal of Industrial Economics, XLI, 77-90.
Eaton, B.C. and N. Schmitt (1994) Flexible manufacturing and market structure, American Economic Review, 84, 875-888.
Hamilton, J., J.-F. Thisse and A. Weskamp (1989) Spatial discrimination: Bertrand vs. Cournot in a model of location choice, Regional Science and Urban Economics, 19, 87-102.
Hotelling, H. (1929) Stability in competition, Economic Journal, 39, 41-57.
MacLeod, W.B., G. Norman and J.-F. Thisse (1988) Price discrimination and equilibrium in monopolistic competition, International Journal of Industrial Organization, 6, 429-446.
Mansfield, E. (1993) The diffusion of flexible manufacturing techniques in Japan, Europe and the United States, Management Science, 39, 149-159.
Milgrom, P. and J. Roberts (1990) The economics of modern manufacturing products, technology and organization, American Economic Review, 80, 511-528.
Motta, M. and G. Norman (1996) Does economic integration cause foreign direct investment? International Economic Review, 37, 757-783.
Norman, G. and J.-F. Thisse (1999) Technology choice and market structure: strategic aspects of flexible manufacturing, Journal of Industrial Economics, XLVII, 345-372.
Röller, L.-H. and M.H. Tombak (1990) Strategic choice of flexible manufacturing technology and welfare implications, The Journal of Industrial Economics, XXXV, 417-431.
Röller, L.-H. and M.H. Tombak (1993) Competition and investment in flexible technologies, Management Science, 39, 107-114.
Rowthorn, R. (1992) Intra-industry trade and investment under oligopoly: the role of market size, Economic Journal, 102, 402-414.
U.S. Office of Technology Assessment (1984), Computerized manufacturing Automation: Employment, Education and the Workplace, Government Printing Office, Washington, D.C.


Figure 1: The Market


Figure 2: Three-stage Perfect Nash Equilibrium -- $k=1$.
Notes:

$$
\begin{aligned}
& \mathbf{c}_{\mathbf{1}}: f(0)=\left(8 \rho-8 \rho^{2}\right) / 9(2-k) ; \mathbf{c}_{\mathbf{2}}: f(0)=\left(16 \rho-24 \rho^{2}\right) / 9(2-k) ; \\
& \mathbf{c}_{\mathbf{3}}: f(0)=16 \rho / 9(2-k) ; \mathbf{b}_{\mathbf{1}}: f(0)=\left(8 \rho-16 \rho^{2}\right) / 9 ; \\
& \mathbf{b}_{\mathbf{2}}: f(0)=\left(8 \rho-12 \rho^{2}\right) / 9 ; \mathbf{b}_{\mathbf{3}}: f(0)=\left(8 \rho-4 \rho^{2}\right) / 9 ; \\
& \mathbf{b}_{\mathbf{4}}: f(0)=\left(8 \rho+8 \rho^{2}\right) / 9 .
\end{aligned}
$$

Table 1: Pay-Off Matrix for $\{p, p\}$ Technology Configuration

|  |  | Firm 1 |  |
| :---: | :---: | :---: | :---: |
|  |  | $\mathbf{l}_{\mathbf{1}}(\{\mathbf{p}, \mathbf{p}\})=\left(0, p_{1}^{2}, 0, d_{1}^{4}, d_{1}^{5}\right)$ | $\mathbf{l}_{\mathbf{1}}(\{\mathbf{p}, \mathbf{p}\})=\left(d_{1}^{1}, 0, p_{1}^{3}, 0, d_{1}^{5}\right)$ |
| $\begin{gathered} \text { Firm } \\ 2 \end{gathered}$ | $\begin{gathered} \mathbf{l}_{2}(\{\mathbf{p}, \mathbf{p}\})= \\ \left(d_{2}^{1}, d_{2}^{2}, 0, p_{2}^{4}, 0\right) \end{gathered}$ | $\begin{aligned} & s \cdot a^{2}\left(\frac{5-4 \rho+6 \rho^{2}}{9}-2 f(0)-f(1)\right) \\ & \text { s. } a^{2}\left(\frac{5-4 \rho+6 \rho^{2}}{9}-2 f(0)-f(1)\right) \end{aligned}$ | $\begin{aligned} & \text { s. } a^{2}\left(\frac{5-4 \rho+10 \rho^{2}}{9}-2 f(0)-f(1)\right) \\ & \text { s. } a^{2}\left(\frac{5-4 \rho+10 \rho^{2}}{9}-2 f(0)-f(1)\right) \end{aligned}$ |
|  | $\begin{gathered} \mathbf{l}_{\mathbf{2}}(\{\mathbf{p}, \mathbf{p}\})= \\ \left(d_{2}^{1}, 0, p_{2}^{3}, 0, d_{2}^{5}\right) \end{gathered}$ | $\begin{aligned} & \text { s. } a^{2}\left(\frac{\frac{5-4 \rho+10 \rho^{2}}{9}-2 f(0)-f(1)}{9}\right\} \text { s. } a^{2}\left(\frac{5-4 \rho+10 \rho^{2}}{9}-2 f(0)-f(1)\right) \end{aligned}$ | $\begin{aligned} & \text { s. } a^{2}\left(\frac{5-4 \rho+2 \rho^{2}}{9}-2 f(0)-f(1)\right. \\ & \text { s. } a^{2}\left(\frac{5-4 \rho+2 \rho^{2}}{9}-2 f(0)-f(1)\right) \end{aligned}$ |

Table 2: Gross Profits to Firm 1 with Technology Choice $\{p, p\}$

| Consumer <br> Submarket | Location Configuration |  |
| :---: | :---: | :---: |
|  | $\left(0, p_{1}^{2}, 0, d_{1}^{4}, d_{1}^{5}\right)$ | $\left(d_{1}^{1}, 0, p_{1}^{3}, 0, d_{1}^{5}\right)$ |
| 1 | $s \cdot a^{2}(1-2 \rho)^{2} / 9$ | $s \cdot a^{2} / 9$ |
| 2 | $s \cdot a^{2}(1+\rho)^{2} / 9$ | $s \cdot a^{2}(1-\rho)^{2} / 9$ |
| 3 | $s \cdot a^{2}(1-2 \rho)^{2} / 9$ | $s \cdot a^{2} / 9$ |
| 4 | $s \cdot a^{2}(1+\rho)^{2} / 9$ | $s \cdot a^{2}(1-\rho)^{2} / 9$ |
| 5 | $s \cdot a^{2} / 9$ | $s \cdot a^{2} / 9$ |

Table 3: Pay-Off Matrix for $\{\mathbf{f}, \mathrm{f}\}$ Technology Configuration

|  |  | Firm 1 |  |
| :---: | :---: | :---: | :---: |
|  |  | $\mathbf{l}_{1}(\{\mathbf{f}, \mathbf{f}\})=\left(0, f_{1}^{2}, 0,0, d_{1}^{5}\right)$ | $\mathbf{l}_{\mathbf{1}}(\{\mathbf{f}, \mathbf{f}\})=\left(0,0, f_{1}^{3}, 0,0\right)$ |
| $\begin{gathered} \text { Firm } \\ 2 \end{gathered}$ | $\begin{gathered} \mathbf{l}_{2}(\{\mathbf{f}, \mathbf{f}\})= \\ \left(d_{2}^{1}, 0,0, f_{2}^{4}, 0\right) \end{gathered}$ | $\begin{aligned} & 1: s . a^{2}\left(\frac{5-8 \rho+26 \rho^{2}}{9}-f(0)-f(2)\right. \\ & 2: s \cdot a^{2}\left(\frac{5-8 \rho+26 \rho^{2}}{9}-f(0)-f(2)\right) \end{aligned}$ | $\begin{gathered} 1: s \cdot a^{2}\left(\frac{5-16 \rho+30 \rho^{2}}{9}-f(2)\right) \\ 2: s \cdot a^{2}\left(\frac{5-4 \rho+18 \rho^{2}}{9}-f(0)-f(2)\right) \end{gathered}$ |
|  | $\begin{gathered} \mathbf{l}_{2}(\{\mathbf{f}, \mathbf{f}\})= \\ \left(0,0, f_{2}^{3}, 0,0\right) \end{gathered}$ | $\begin{gathered} 1: s \cdot a^{2}\left(\frac{5-4 \rho+18 \rho^{2}}{9}-f(0)-f(2)\right) \\ \quad 2: s \cdot a^{2}\left(\frac{5-16 \rho+30 \rho^{2}}{9}-f(2)\right) \end{gathered}$ | $\begin{aligned} & \text { 1:s. } a^{2}\left(\frac{5-12 \rho+10 \rho^{2}}{9}-f(2)\right. \\ & \text { 2: s. } a^{2}\left(\frac{5-12 \rho+10 \rho^{2}}{9}-f(2)\right) \end{aligned}$ |

Table 4: Profits in the Technology Configuration $\{d, f\}$

| Location <br> Configuration of Firm <br> 2 | Profit of Firm 2 | Profit of Firm 1 |
| :---: | :---: | :---: |
| $\left(d_{2}^{1}, 0,0, f_{2}^{4}, 0\right)$ | $s . a^{2}\left(\frac{5-16 \rho+24 \rho^{2}}{9}-f(0)-f(2)\right)$ | $s . a^{2}\left(\frac{5+8 \rho+6 \rho^{2}}{9}-5 f(0)\right)$ |
| $\left(0,0, f_{2}^{3}, 0,0\right)$ | $s . a^{2}\left(\frac{5-24 \rho+40 \rho^{2}}{9}-f(2)\right)$ | $s . a^{2}\left(\frac{5+12 \rho+10 \rho^{2}}{9}-5 f(0)\right)$ |

Table 5: Gross Profits to Firm 2 with Technology Choice $\{d, f\}$

| Consumer <br> Submarket | Location Configuration |  |
| :---: | :---: | :---: |
|  | $\left(d_{2}^{1}, 0,0, f_{2}^{4}, 0\right)$ | $\left(0,0, f_{2}^{3}, 0,0\right)$ |
| 1 | $s \cdot a^{2} / 9$ | $s \cdot a^{2}(1-4 \rho)^{2} / 9$ |
| 2 | $s \cdot a^{2}(1-4 \rho)^{2} / 9$ | $s \cdot a^{2}(1-2 \rho)^{2} / 9$ |
| 3 | $s \cdot a^{2}(1-2 \rho)^{2} / 9$ | $s \cdot a^{2} / 9$ |
| 4 | $s \cdot a^{2} / 9$ | $s \cdot a^{2}(1-2 \rho)^{2} / 9$ |
| 5 | $s \cdot a^{2}(1-2 \rho)^{2} / 9$ | $s \cdot a^{2}(1-4 \rho)^{2} / 9$ |

Table 6: Pay-Off Matrix for $\{\mathbf{p}, \mathbf{f}\}$ Technology Configuration

|  |  | Firm 2 |  |
| :---: | :---: | :---: | :---: |
|  |  | $\mathbf{I}_{2}(\{\mathbf{p}, \mathbf{f}\})=\left(d_{2}^{1}, 0,0, f_{2}^{4}, 0\right)$ | $\mathbf{I}_{2}(\{\mathbf{p}, \mathbf{f}\})=\left(0,0, f_{2}^{3}, 0,0\right)$ |
| Firm <br> 1 | $\begin{gathered} \mathbf{l}_{1}(\{\mathbf{p}, \mathbf{f}\})= \\ \left(0, p_{1}^{2}, 0, d_{1}^{4}, d_{1}^{5}\right) \end{gathered}$ | $\begin{gathered} 1: s . a^{2}\left(\frac{5-12 \rho+22 \rho^{2}}{9}-f(0)-f(2)\right. \\ 2: s . a^{2}\left(\frac{5+10 \rho^{2}}{9}-2 f(0)-f(1)\right) \end{gathered}$ | $\begin{gathered} 1: s . a^{2}\left(\frac{5-20 \rho+34 \rho^{2}}{9}-f(2)\right) \\ 2: s . a^{2}\left(\frac{5+4 \rho+10 \rho^{2}}{9}-2 f(0)-f(2)\right) \end{gathered}$ |
|  | $\begin{gathered} \mathbf{l}_{\mathbf{1}}(\{\mathbf{p}, \mathbf{f}\})= \\ \left(d_{1}^{2}, 0, p_{1}^{3}, 0, d_{1}^{5}\right) \end{gathered}$ | $\begin{aligned} & 1: s . a^{2}\left(\frac{5-12 \rho+18 \rho^{2}}{9}-f(0)-f(2)\right. \\ & 2: s . a^{2}\left(\frac{5-2 \rho+7 \rho^{2}}{9}-2 f(0)-f(1)\right) \end{aligned}$ | $\begin{gathered} 1: s . a^{2}\left(\frac{5-20 \rho+34 \rho^{2}}{9}-f(2)\right) \\ 2: s . a^{2}\left(\frac{5+4 \rho+10 \rho^{2}}{9}-2 f(0)-f(1)\right) \end{gathered}$ |
|  | $\begin{gathered} \mathbf{I}_{\mathbf{1}}(\{\mathbf{p}, \mathbf{f}\})= \\ \left(d_{1}^{2}, d_{1}^{2}, 0, p_{1}^{4}, 0\right) \end{gathered}$ | $\begin{gathered} 1: s . a^{2}\left(\frac{5-12 \rho+18 \rho^{2}}{9}-f(0)-f(2)\right) \\ 2: s . a^{2}\left(\frac{5+6 \rho^{2}}{9}-2 f(0)-f(1)\right) \end{gathered}$ | $\begin{gathered} 1: s . a^{2}\left(\frac{5-20 \rho+34 \rho^{2}}{9}-f(2)\right) \\ 2: s . a^{2}\left(\frac{5+4 \rho+10 \rho^{2}}{9}-2 f(0)-f(1)\right) \end{gathered}$ |

Table A1a: Technology Choice Pay-Off Matrix - $f(0) \leq\left(8 \rho-16 \rho^{2}\right) / 9$

|  |  | Firm 1 |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | d | p | f |
| Firm 2 | d | $\begin{aligned} & \text { 1: } 5 s \cdot a^{2}(1 / 9-f(0)) \\ & \text { 2: } 5 s \cdot a^{2}(1 / 9-f(0)) \end{aligned}$ | $\begin{gathered} \text { 1: } s . a^{2}\left(\left(5-8 \rho+8 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: s. } a^{2}\left(\left(5+4 \rho+2 \rho^{2}\right) / 9\right. \\ -5 f(0)) \end{gathered}$ | $\begin{gathered} \text { 1: } s . a^{2}\left(\left(5-16 \rho+24 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \text { 2: s. } a^{2}\left(\left(5+8 \rho+6 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \hline \end{gathered}$ |
|  | p | $\begin{gathered} \text { 1: } s . a^{2}\left(\left(5+4 \rho+2 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \text { 2: s. } a^{2}\left(\left(5-8 \rho+8 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{gathered}$ | $\begin{array}{r} \text { 1: } s . a^{2}\left(\left(5-4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: s. } a^{2}\left(\left(5-4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{array}$ | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-12 \rho+22 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \text { 2: s. } a^{2}\left(\left(5+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{gathered}$ |
|  | f | $\begin{gathered} \text { 1: } s . a^{2}\left(\left(5+8 \rho+6 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \text { 2: s. } a^{2}\left(\left(5-16 \rho+24 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \hline \hline \end{gathered}$ | $\begin{gathered} \text { 1: } s \cdot a^{2}\left(\left(5+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: } s . a^{2}\left(\left(5-12 \rho+22 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \hline \hline \end{gathered}$ | $\begin{array}{r} \text { 1: } s \cdot a^{2}\left(\left(5-8 \rho+26 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \text { 2: } s . a^{2}\left(\left(5-8 \rho+26 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \hline \hline \end{array}$ |

Table A1b: Technology Choice Pay-Off Matrix $-\left(8 \rho-16 \rho^{2}\right) / 9<f(0) \leq\left(8 \rho-12 \rho^{2}\right) / 9$

|  |  |  | Firm 1 |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | d | p | f |
| Firm 2 | d | $\begin{aligned} & \text { 1: } 5 s \cdot a^{2}(1 / 9-f(0)) \\ & \text { 2: } 5 s \cdot a^{2}(1 / 9-f(0)) \end{aligned}$ | $\begin{gathered} \text { 1: } s \cdot a^{2}\left(\left(5-8 \rho+8 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: } s \cdot a^{2}\left(\left(5+4 \rho+2 \rho^{2}\right) / 9\right. \\ -5 f(0)) \end{gathered}$ | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-24 \rho+40 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \text { 2: s. } a^{2}\left(\left(5+12 \rho+10 \rho^{2}\right) / 9\right. \\ -5 f(0)) \end{gathered}$ |
|  | p | $\begin{gathered} \text { 1: } s . a^{2}\left(\left(5+4 \rho+2 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \text { 2: } s \cdot a^{2}\left(\left(5-8 \rho+8 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \end{gathered}$ | $\begin{array}{r} \text { 1: } s \cdot a^{2}\left(\left(5-4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: } s \cdot a^{2}\left(\left(5-4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{array}$ | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-12 \rho+22 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \text { 2: s. } a^{2}\left(\left(5+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{gathered}$ |
|  | f | $\begin{gathered} \text { 1: } s \cdot a^{2}\left(\left(5+12 \rho+10 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \text { 2: s. } a^{2}\left(\left(5-24 \rho+40 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \end{gathered}$ |  | $\begin{array}{r} \text { 1: } s . a^{2}\left(\left(5-8 \rho+26 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \text { 2: } s a^{2}\left(\left(5-8 \rho+26 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \hline \hline \end{array}$ |

Table A1c: Technology Choice Pay-Off Matrix - $\left(8 \rho-12 \rho^{2}\right) / 9 \leq f(0)<\left(8 \rho-4 \rho^{2}\right) / 9$

|  |  | Firm 1 |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | d | p | f |
| Firm 2 | d | $\begin{aligned} & \text { 1: } 5 s \cdot a^{2}(1 / 9-f(0)) \\ & \text { 2: } 5 s \cdot a^{2}(1 / 9-f(0)) \end{aligned}$ | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-8 \rho+8 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: } s \cdot a^{2}\left(\left(5+4 \rho+2 \rho^{2}\right) / 9\right. \\ -5 f(0)) \end{gathered}$ | $\begin{gathered} \text { 1: } s . a^{2}\left(\left(5-24 \rho+40 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \text { 2: } s . a^{2}\left(\left(5+12 \rho+10 \rho^{2}\right) / 9\right. \\ -5 f(0)) \end{gathered}$ |
|  | p | $\begin{gathered} \text { 1: } s . a^{2}\left(\left(5+4 \rho+2 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \text { 2: } s . a^{2}\left(\left(5-8 \rho+8 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \end{gathered}$ | $\begin{array}{r} \text { 1: } s \cdot a^{2}\left(\left(5-4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: } s \cdot a^{2}\left(\left(5-4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{array}$ | $\begin{gathered} \text { 1: } s \cdot a^{2}\left(\left(5-20 \rho+34 \rho^{2}\right) / 9\right. \\ -f(2)) \\ \text { 2: } s \cdot a^{2}\left(\left(5+4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{gathered}$ |
|  | f | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5+12 \rho+10 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \text { 2: } s . a^{2}\left(\left(5-24 \rho+40 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \hline \hline \end{gathered}$ | $\begin{gathered} \text { 1: } s \cdot a^{2}\left(\left(5+4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: } s \cdot a^{2}\left(\left(5-20 \rho+34 \rho^{2}\right) / 9\right. \\ -f(2)) \\ \hline \hline \end{gathered}$ | $\begin{array}{r} \text { 1: } s \cdot a^{2}\left(\left(5-8 \rho+26 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \text { 2: } s a^{2}\left(\left(5-8 \rho+26 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \hline \hline \end{array}$ |

Table A1d: Technology Choice Pay-Off Matrix - $\left(8 \rho-4 \rho^{2}\right) / 9<f(0) \leq\left(8 \rho+8 \rho^{2}\right) / 9$

|  |  | Firm 1 |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | d | p | f |
| Firm 2 | d | $\begin{aligned} & \text { 1: } 5 s \cdot a^{2}(1 / 9-f(0)) \\ & \text { 2: } 5 \cdot a^{2}(1 / 9-f(0)) \end{aligned}$ | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-8 \rho+8 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: s. } a^{2}\left(\left(5+4 \rho+2 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \hline \end{gathered}$ | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-24 \rho+40 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \text { 2: } s . a^{2}\left(\left(5+12 \rho+10 \rho^{2}\right) / 9\right. \\ -5 f(0)) \end{gathered}$ |
|  | p | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5+4 \rho+2 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \text { 2: s. } a^{2}\left(\left(5-8 \rho+8 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{gathered}$ | $\begin{array}{r} \text { 1: s. } a^{2}\left(\left(5-4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: s. } a^{2}\left(\left(5-4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{array}$ | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-20 \rho+34 \rho^{2}\right) / 9\right. \\ -f(2)) \\ \text { 2: } s . a^{2}\left(\left(5+4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{gathered}$ |
|  | f | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5+12 \rho+10 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \text { 2: } s . a^{2}\left(\left(5-24 \rho+40 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \hline \hline \end{gathered}$ |  | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-16 \rho+30 \rho^{2}\right) / 9\right. \\ -f(2)) \\ \text { 2: } s . a^{2}\left(\left(5-4 \rho+18 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \hline \hline \end{gathered}$ |

Table A1e: Technology Choice Pay-Off Matrix - $\left(8 \rho+8 \rho^{2}\right) / 9<f(0)$

|  |  | Firm 1 |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | d | p | f |
| Firm 2 | d | $\begin{aligned} & \text { 1: } 5 s \cdot a^{2}(1 / 9-f(0)) \\ & \text { 2: } 5 s \cdot a^{2}(1 / 9-f(0)) \end{aligned}$ | $\begin{gathered} \text { 1: } s \cdot a^{2}\left(\left(5-8 \rho+8 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: s. } a^{2}\left(\left(5+4 \rho+2 \rho^{2}\right) / 9\right. \\ -5 f(0)) \end{gathered}$ | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-24 \rho+40 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \text { 2: } s . a^{2}\left(\left(5+12 \rho+10 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \hline \end{gathered}$ |
|  | p | $\begin{gathered} \text { 1: } s . a^{2}\left(\left(5+4 \rho+2 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \text { 2: } s \cdot a^{2}\left(\left(5-8 \rho+8 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{gathered}$ | $\begin{array}{r} \text { 1: } s \cdot a^{2}\left(\left(5-4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: } s \cdot a^{2}\left(\left(5-4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{array}$ | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-20 \rho+34 \rho^{2}\right) / 9\right. \\ -f(2)) \\ \text { 2: } s \cdot a^{2}\left(\left(5+4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \hline \end{gathered}$ |
|  | f | $\begin{gathered} \text { 1: } s . a^{2}\left(\left(5+12 \rho+10 \rho^{2}\right) / 9\right. \\ -5 f(0)) \\ \text { 2: s. } a^{2}\left(\left(5-24 \rho+40 \rho^{2}\right) / 9\right. \\ -f(0)-f(2)) \\ \hline \hline \end{gathered}$ | $\begin{gathered} \text { 1: } s \cdot a^{2}\left(\left(5+4 \rho+10 \rho^{2}\right) / 9\right. \\ -2 f(0)-f(1)) \\ \text { 2: } s \cdot a^{2}\left(\left(5-20 \rho+34 \rho^{2}\right) / 9\right. \\ -f(2)) \\ \hline \hline \end{gathered}$ | $\begin{gathered} \text { 1: s. } a^{2}\left(\left(5-12 \rho+10 \rho^{2}\right) / 9\right. \\ -f(2)) \\ \text { 2: s. } a^{2}\left(\left(5-12 \rho+10 \rho^{2}\right) / 9\right. \\ -f(2)) \\ \hline \hline \end{gathered}$ |


[^0]:    3 We do not consider technologies with widths greater than 2 given that we confine attention to a market

[^1]:    4 We do not feel that anything is to be gained from distinguishing $\mathbf{p}$ and $\mathbf{f}$ technologies with respect to $r$. $5 \quad$ As an illustration we can consider flexible manufacturing in the production of elevators. Clients are typically offered a wide range of external finishes and car sizes -- the parts that users see -- but a much smaller range of drive speeds and capacities, control systems and other components -- that the users do not see. Custom-built elevators, by contrast, offer a much wider range of specifications of the total system.

[^2]:    7 Coincident location at $(2,2)$ and $(4,4)$ are equivalent to $(3,3)$.
    8 Note that these are equivalent equilibria.

[^3]:    9 Note that the four market location configurations of Table 1 have identical total set-up costs for each firm.
    10 This is reminiscent of game theoretic models of foreign direct investment in which it is shown that the relative profitability of different location configurations is determined by the balance between the import protection effect of competing with imported products and the export cost effect of having to export a product to another market: see Motta and Norman (1996).

[^4]:    11 Note that these are equivalent equilibria.

[^5]:    comments apply to the discussion of technology configurations $\{\mathbf{f}, \mathbf{d}\}$ and $\{\mathbf{f}, \mathbf{p}\}$ below.

