Should Pricing Policies be Regulated when Firms may Tacitly Collude?

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Abstract:

Active antitrust policy may result in perverse effects detrimental to consumer and social welfare because such active policy affects market structure through its impact on the medium- and long-run decisions of firms. This paper investigates the effects of deregulation of firms' pricing policies in a dynamic setting that allows for tacit collusion. We show that, provided the consumer reservation price imposes no effective constraints on firms, price deregulation will lead to discriminatory prices that are almost always and almost everywhere lower than mill prices. By contrast, if the consumer reservation price is binding, the welfare properties of the two pricing policies will be reversed with consumers losing and firms gaining from discriminatory pricing. The required degree of price flexibility for this to happen is lower in more concentrated market structures. This implies that if market concentration is already high, deregulation of pricing policies is less likely to benefit consumers unless the regulatory authorities are willing to impose accompanying restrictions intended to reduce price flexibility.

Keywords: deregulation, price discrimination, dynamic game
JEL: L10, L13, L50.
1. Introduction

Regulation of the conditions under which firms compete is common in the majority of Western industrialized countries. Such regulation is intended to prevent or control, for example, explicit or tacit collusion among firms, mergers and acquisitions, vertical restraints and firms’ pricing policies, in particular whether firms can employ discriminatory pricing. The common rationale for these regulatory policies is that they are needed to protect consumers from the abuse of monopoly power by firms that supply them with goods and services.

The purpose of this paper is to suggest that care should be exercised by antitrust authorities in their design of policies intended to promote competition in the marketplace. We do not deny the underlying “raison d'être” of competition policy but wish to suggest that naive application of the idea that competition is always and everywhere desirable may have unforeseen and harmful effects. Our analysis can be summarized by the proposition that analysis of the effects of competition policy should not take industry structure as given. Policies that create too tough a competitive environment may result in perverse effects detrimental to consumer and social welfare because active antitrust policy affects market structure through its impact on the medium- and long-run decisions of firms. The stronger are the structural effects of regulatory policy the more likely is it that blind adherence by the regulatory authorities to the benefits of competition will be misguided.

As a simple illustration, consider the case for cartel laws. The received wisdom has been that a high degree of coordination among firms benefits them through high profits but is detrimental to consumers through high prices, the losses of consumers generally outweighing the gains of producers. This line of reasoning underpinned the tough anti-trust and merger policy that characterized regulation in the United States and European until the early 1980s. The desirability and acceptability of such tough policy has, however, been questioned on several grounds. Williamson (1968), for example, has argued that efficiency considerations may be grounds for defending coordination among firms if such coordination allows them to perform cost-saving activities, the benefits of which outweigh the price-increasing or quantity-reducing effects of coordination.¹

¹ German law allows "rationalization" and "specialization" cartels on the grounds that they lead to efficiency gains (see Kühn 1993).
Selten (1984) develops an argument that is more closely related to the ideas we develop in this paper. He shows that if the market within which cartel laws are being enacted is characterized by free entry “joint profit maximization permits a greater number of competitors in a market than non-collusive behavior.” (p. 183) As a consequence “cartel laws are good for business in the sense of greater average joint profits.” (p. 214)

Our focus in this paper is on the regulation of firms’ pricing policies, which was widely accepted up to the early 1980's to be in the public interest. Norman and Thisse (1996) provide several examples of such regulation: of the prices charged by US and European airline industries, the application of resale price maintenance, the policies articulated by the UK Price Commission and the Federal Trade Commission in the US attacking discriminatory pricing. In a related context, the 1980s saw increasingly strict application by the US and Europe of anti-dumping legislation (dumping can be viewed as price discrimination in the international market-place). For example, from 1980 to 1987 the EC ruled against foreign firms in 249 cases affecting some three billion ECU's of imports (Tharakan 1991, Table 1).

This relatively tough regulatory regime has been considerably relaxed over the past decade. The Robinson-Patman Act has not been applied in the US, the Price Commission was abolished in the UK, and more generally firms have been left freer to choose their pricing policies, as a result of which (spatial) discriminatory pricing is a more likely outcome (Thisse and Vives, 1988).

Justification for the relaxation in policy can be found in modern developments in spatial pricing which argue that spatial price discrimination may be better for consumers than mill pricing: see, for example, Norman (1983), Thisse and Vives (1988). The way the argument runs is that discriminatory pricing is tougher for firms than mill pricing and so is pro- rather than anti-competitive:

"denying a firm the right to meet the price of a competitor on a discriminatory basis provides the latter with some protection against price attacks. The effect is then to weaken competition, contrary to the belief of the proponents of naive application of legislation prohibiting price discrimination like the Robinson-Patman Act, or similar recommendations of the Price Commission in the United Kingdom." (Thisse and Vives, 1988)
It is of interest to note, for example, that discriminatory prices can be defended against action under the Robinson-Patman Act or international anti-dumping legislation if they are intended to "meet the competition".

The benefits claimed for discriminatory pricing have been called into question in recent analyses by Armstrong and Vickers (1993) and Norman and Thisse (1996). The common theme of these works is that current policy-based analysis is flawed because it takes no account of an important feedback loop between market conduct and market structure.\footnote{Armstrong and Vickers consider a situation in which a monopolist serves two markets, one of which is captive and the other subject to the threat of entry by a price-taking entrant. They show that entry is more likely and the scale of entry is larger when price discrimination is banned. Norman and Thisse consider a free-entry Salop-style market and show that price discrimination may inhibit entry sufficiently as to be against the interests of consumers and society. See d'Aspremont and Motta (1997) for a model in a related vein.} If discriminatory pricing is, indeed, more competitive for incumbent firms it will act as an entry deterrent. As a result, the essentially short-run benefits that the more competitive regime generates through lower prices may be more than offset by the longer-run effects it has on market structure: by reducing in the number of firms that wish to enter the market or the scale at which they enter.

Our specific contribution in this paper is that we extend the Norman/Thisse analysis by relaxing an important assumption: that post-entry prices are one-shot Bertrand equilibrium prices. We consider, instead, the situation in which the incumbent firms recognize that the price game is a repeated game. Before turning to this analysis, we develop some further preliminary ideas in the next section. Our formal model is then presented in section 3 and the price equilibria for this model are identified in section 4. Section 5 presents a welfare comparison of the two pricing policies and our main conclusions are summarized in section 6.

2. Some Preliminary Analysis

Let us try to make the ideas presented above more concrete by examining how we might structure a general case capable of identifying the supposed benefits of tough regulatory policies. Assume that a market (as defined for policy purposes) contains \( N \) active firms, where \( N \) is to be determined endogenously through some entry and/or exit process. The pay-off to each firm is a function of two sets of variables:

(i) a set \( R \) describing the regulatory regime within which the firms operate. \( R \) may be determined by Government, for example describing the class of acceptable pricing policies or the toughness of antitrust policy, or by (a subset of) the incumbent firms, for
example, specifying “orderly” pricing regimes such as basing point pricing; consumer incentives; price-matching guarantees.

(ii) a set \( P \) of strategic variables chosen by the firms more or less non-cooperatively: prices, product specifications; locations; quantities.

A subgame-perfect Nash equilibrium in \( N \) and \( P \) is identified from some \( m \)-stage game. For a given regulatory regime \( R \), this equilibrium specifies a pay-off to each firm of the form:

\[
\Pi_i^* \left( P^* \left( N^* \right), N^*:R \right) \text{ for } i = 1, \ldots, N^* \]

which will have associated welfare properties. Now consider alternative regulatory regimes, where we define a “tougher” regime \( \hat{R} > R \) as being a regime that is more competitive for the firms in, or contemplating entry to the market. For example, assume that the regulatory authority tries to promote fierce competition in this market by abolishing facilitating practices, or outlawing the trade association to avoid contacts among the firms, or warning the firms that they are suspected of collusive behavior. For a given \( N \) it is to be expected that:

\[
\Pi_i^* \left( P^* \left( N \right), N: \hat{R} \right) < \Pi_i^* \left( P^* \left( N \right), N: R \right) \text{ if } \hat{R} > R
\]

with the reduction in profit (partially) transferred to consumers. Equation (2) would appear to justify governments’ preference for tough rather than soft regulatory regimes.

However, equation (2) is essentially short-run, in that it takes market structure as given. It is also to be expected that increased toughness of the regulatory regime will reduce the number of active firms in the market in the medium- and long-run. When firms have to decide whether to renew plants or to update their products or to incur other fixed costs, it may be that the decreased future stream of expected profits will not cover these costs. The exit of some firms is to be expected, increasing profitability for the remaining firms in the market: \(^3\)

\[
\Pi_i^* \left( P^* \left( N_1 \right), N_1: R \right) < \Pi_i^* \left( P^* \left( N_2 \right), N_2: R \right) \text{ if } N_1 > N_2
\]

The overall effect of an increase in the toughness of the regulatory regime is a combination of (2) and (3) and might be expressed as follows. Let \( \Delta R > 0 \) be some measure of increased toughness of \( R \). Then

\[
\frac{\Delta \Pi_i}{\Delta R}_{|_N} = \frac{\Delta \Pi_i}{\Delta P} \frac{\Delta P}{\Delta R} < 0 \text{ and } \frac{\Delta \Pi_i}{\Delta N}_{|_R} < 0 \text{ but}
\]
\[
(4) \quad \frac{\Delta \Pi}{\Delta R} = \frac{\Delta \Pi}{\Delta P} \frac{\Delta P}{\Delta R} + \left[ \frac{\Delta \Pi}{\Delta N} \frac{\Delta N}{\Delta P} + \frac{\Delta \Pi}{\Delta \Pi} \frac{\Delta \Pi}{\Delta N} \right] \frac{\Delta N}{\Delta R} = ? .
\]

In other words, increased toughness of \( R \) has:

(a) a profit reducing effect in the short run through its impact on \( P \) for a given \( N \), and

(b) a profit increasing effect in the longer run through induced exit and stronger entry deterrence.

For increased toughness to benefit consumers (a) must dominate (b). Otherwise increased market concentration will make consumers worse off, for example through reduced product variety or a longer travel distance to shops, and more generally through higher prices. Note the similarity between this analysis and Selten’s argument that collusion may lead to lower concentration and lower total profits in an industry. A qualitatively similar conclusion would be reached if our measure of the desirability of changes in \( R \) were some weighted average of producer and consumer surplus.\(^4\) In short, policy that establishes too tough a competitive environment may have some unforeseen effects with the (apparently) paradoxical result of being detrimental to the community's welfare.

An important limitation of almost all current analysis is that the equilibrium of the post-entry subgame, on whatever strategic variables it is based, is assumed to be the outcome of a one-shot game. This, as we shall show, is an important assumption. If price, for example, is the strategic variable, discriminatory pricing is, indeed, more competitive than non-discriminatory (referred to hereafter as mill) pricing in spatial markets, or their non-spatial analogs, precisely because the prices that emerge are the result of a Bertrand-at-every point process. But if the price subgame is repeated it is not at all clear that the same conclusion will hold.

We know that a monopolist always prefers discriminatory pricing to mill pricing and it might be expected that the same will be true when the prices charged by firms correspond to a tacitly collusive outcome for an infinitely repeated price subgame.\(^5\) Assume, for example, as we do in the remainder of our analysis, the familiar Hotelling/Salop spatial model in which consumer demand is perfectly inelastic for prices below some reservation price. With demand of

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\(^3\) A possible exception would be if there were extensive network externalities between firms.

\(^4\) There are further considerations that we do not consider in this paper. For example, price discrimination facilitates market segmentation by multi-product firms, increasing prices and profits but reducing consumer surplus.

\(^5\) We can assume the price subgame to be infinitely repeated or to be repeated with some known, constant probability in each period \( t \) that the subgame will continue.
this type it trivially follows from the Folk Theorem that a set of discriminatory prices arbitrarily close to the consumers’ reservation price will be subgame perfect equilibrium (SPE) prices for some discount factor sufficiently close to unity. Consequently, these prices will be higher than the SPE mill prices almost everywhere even if market structure is endogenously determined.

It would appear, in other words, that the “endogenous market structure” argument is irrelevant to the choice of allowable pricing practices once we introduce the possibility that firms are aware that they are in competition over time and incorporate this into their pricing decisions. But this overlooks the importance of a crucial part of the argument: that the discount factor be “sufficiently close to unity”. The question we need to investigate is how close the discount factor must be to unity for a particular set of discriminatory or mill prices to be sustainable as an SPE. This question arises from the nature of SPE prices. These are sustained through some credible punishment strategy which is designed to ensure that the short-term gains from deviating from the (tacitly) agreed prices are more than offset by the long-term losses that will be incurred once the punishment phase is implemented. Looking at this another way, the ability to raise prices above their Bertrand-Nash equilibrium levels is constrained by the potential gains a firm will make by deviating from these prices. Such gains are likely to vary significantly with different pricing policies and so will impose different limits on the SPE prices. This suggests that we might approach the problem from a somewhat different perspective by asking: For a given discount factor, what is the upper limit on the discriminatory and mill prices that firms can charge? We can then investigate the welfare properties of these prices.

Two other remarks are in order at this stage. First, the punishment strategy on which we focus in this paper is what is sometimes referred to as a "grim trigger strategy": any deviation from the tacitly agreed prices induces reversion to the one-shot Bertrand-Nash equilibrium prices forever. Other punishments are available to firms but it is reasonable to suggest that the qualitative properties of our analysis would remain valid for a larger class of such alternative strategies.

Secondly, it will be shown below that the discount factor is an amalgam of the discount rate, which is largely outside the control of regulators (and firms), and firms' price flexibility in response to deviation by their rivals, which is potentially controllable by regulators. Under what circumstances (if any) might the authorities wish to impose controls on the speed with which
firms can change their prices? Our analysis, in other words, introduces a further important element to regulatory control that has not been much discussed, primarily because the currently available analysis is static. The dynamic approach followed here brings with it a new dimension of competition in that the discount factor can be made endogenous to the market.

We show that, provided consumers’ reservation prices impose no effective constraints on firms, the upper limits on the tacitly agreed prices are such that discriminatory prices are almost always and almost everywhere lower than mill prices. In other words, consumers gain and firms lose from discriminatory pricing. The ease with which firms can revise their location decisions (the degree of spatial contestability), which was central to the welfare properties of our 1996 analysis, now has a more limited role. Firms gain less and consumers lose less from mill pricing when the market is spatially non-contestable.

We also show, however, that explicit account must be taken of the consumer reservation price since this imposes a more severe constraint on mill prices than on discriminatory prices. If this constraint is strong enough and if pricing decisions are flexible enough, the welfare properties of the two pricing policies will be reversed with consumers losing and firms gaining from discriminatory pricing. The required degree of price flexibility for this to happen is lower in more concentrated market structures. This implies that if market concentration is already high, deregulation of pricing policies is less likely to benefit consumers unless the regulatory authorities are willing to impose accompanying restrictions intended to reduce price flexibility.

3. The Model

We conduct our analysis in the context of the Hotelling/Salop location model, in which the economy is assumed to be a circle $C$ over which consumers are uniformly distributed at unit density. The interpretation of this model as a model of horizontal product differentiation is now so familiar that we need not repeat it here.

We assume that each time period $t$ is $\gamma$ units long, where $\gamma$ measures the speed of response of incumbent firms to any perceived price deviation, and that the continuous rate of time discount is $\rho$. Firms entering this market offer products that are identical in all characteristics but their locations. Production costs in any period $t$ are assumed to be identical for all firms, given by:

\[
C'(Q') = f + cQ'
\]
where \( f \) can be thought of as the flow-equivalent present value of set-up costs -- if aggregate set-up costs are \( F \), then 
\[
f = F \int_{0}^{\infty} \int_{0}^{t} e^{-\rho \tau} d\tau = F / \rho
\]
-- \( c \) are marginal costs and \( Q^{t} \) is total output of the firm in period \( t \). Thus production exhibits economies of scale, limiting the number of firms that will wish to enter the market. We assume that \( f \) is sufficiently small for us to be able to ignore integer problems and we normalize \( c = 0 \) without loss of generality.

The set \( N = \{1, 2, \ldots, n, \ldots\} \) denotes all potential entrants and the set \( \mathbb{K} \) denotes the firms that actually choose to enter.\(^6\) Each firm chooses a location \( x_{i} \in C \cup \Phi \), where \( x_{i} \in C \) if firm \( i \) chooses to enter the market and \( x_{i} = \Phi \) otherwise. This entry stage establishes a location configuration denoted by the vector \( x = (x_{i}) \) in which we assume that the set \( \mathbb{K} \) corresponds to the first \( \#(\mathbb{K}) \) elements of \( x \). It will prove convenient below to refer to the time-dependent location configuration \( x^{t} \), where, unless otherwise stated, it is to be expected that \( x^{t} = x \) for all \( t = 1, \ldots, \infty \).

Firms are assumed to make their pricing decisions after they have made their entry/location decisions. In each post-entry period \( t \), each firm \( i \in \mathbb{K} \) establishes the price schedule \( p^{t}_{i}(r|x^{t}) \) giving its delivered price to each consumer location \( r \in C \). Transport costs are assumed to be linear in distance and quantity transported with transport rate \( s \), so that with mill pricing the delivered price schedule for firm \( i \) in period \( t \) is:

\[
6 \quad p^{t}_{i}(r|x^{t}) = m^{t}_{i} + s \|x_{i} - r\| \quad \forall \, i \in \mathbb{K}, r \in C.
\]

With discriminatory pricing we assume only that firms never price below marginal costs:

\[
7 \quad p^{t}_{i}(r|x^{t}) \geq s \|x_{i} - r\| \quad \forall \, i \in \mathbb{K}, r \in C
\]

and we normalize \( s = 1 \) without loss of generality. The firms’ pricing decisions made in each period are denoted by the vector \( p^{t}(x^{t}) = \{ i \{x^{t}\} \} \), and \( P^{t}(x^{t}), i \in \mathbb{K} \), denotes the set of feasible delivered price schedules for each entrant in period \( t \). Let \( s^{t} = (x^{t}, p^{t}(x^{t})) \) and \( s = \{s^{0}, s^{1}, \ldots, s^{t}, \ldots\} \).

Consumers purchase from the firm offering the product at the lowest delivered price. If there is a price tie, we assume that consumers act in a socially optimal manner by purchasing

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\(^6\) This implies that all firms make their entry decisions simultaneously. In the analysis that follows we shall concentrate upon entry processes that lead to the loosest packing of firms consistent with the relocation costs that
from the nearest firm. Consumer demand is perfectly inelastic at prices below a reservation price \( v \), with demand to firm \( i \in \mathbb{K} \) from consumers at location \( r \) given by:

\[
q_i'(s^t) = \begin{cases} 
1 & \text{if } p_i'(r|x^t) \leq v \text{ and } p_i'(r|x^t) = \min_j p_j'(r|x^t) \\
0 & \text{otherwise.}
\end{cases}
\]

The per-period rate of profit to firm \( i \in \mathbb{K} \) is:

\[
\pi_i'(s^t) = \int_c \int_{r_i}^{(r+1)^\gamma} [p_i'(r|x^t) - \|x_i - r\|] q_i'(s^t) e^{-\rho t} d\tau dr - f \int_{r_i}^{(r+1)^\gamma} e^{-\rho t} d\tau
= \frac{1 - \delta}{\rho} \delta' \left[ \int_c [p_i'(r|x^t) - \|x_i - r\|] q_i'(s^t) dr - f \right]
\]

where \( \delta = e^{-\gamma \rho} \) is the *discount factor*, with \( 0 \leq \delta \leq 1 \). The discounted present value of firm \( i \) is:

\[
V_i(s) = \sum_{t=0}^{\infty} \pi_i'(s^t).
\]

Since the discount factor \( \delta \) is determined by both \( \gamma \) and \( \rho \) it can be taken as a measure of price flexibility in the relevant market which, as we noted in section 2, is potentially controllable by the regulatory authorities. In other words, the discount factor can be made endogenous to the market by the regulator.

In analyzing equilibrium in this market we distinguish between the initial entry stage and subsequent periods in which incumbent firms can take explicit account of the repeated nature of price competition between them. Since in these later periods “profits will normally be shared on the basis of market areas ... we may expect the process by which market areas are determined to be a very aggressive one.” (MacLeod *et al.*, 1987, p. 192). Specifically, we assume that market areas are determined by a two-stage entry/price game in which equilibrium for the price-subgame is Bertrand-Nash. This is defined in the usual way as the set of price functions \( p'_b(x^t) \) such that for all \( i \in \mathbb{K} \):

\[
\pi_i'(x^t, p'_b(x^t)) \geq \pi_i'(x^t, p'_i(x^t), p'_{-ib}(x^t)) \quad \forall p'_i(x^t) \in P'_i(x^t).
\]

Denote \( s'_b = (x^t, p'_b(x^t)) \).

Equilibrium for the first stage entry game is a location configuration \( x \) such that:
We follow a well-established tradition and confine our attention to location configurations in which incumbent firms are symmetrically located on $C$.

Once firms’ market areas have been determined by this entry/price process the identities of the incumbent firms are known and it becomes possible for them to coordinate their pricing decisions to reflect the repeated nature of the price game in which they are now involved. We can relabel as $t = 0$ the beginning of the post-entry repeated price game. Given the equilibrium $(x, p^*_b(x))$ identified by (11) and (12), it is always possible to find a set of prices $\tilde{p} = \{\tilde{p}_i\}$ for $i \in \mathcal{N}$ such that $\pi_i(x, \tilde{p}) \geq \pi_i(x, p^*_i(x))$. MacLeod et al. (1987) show that the pricing strategy:

$$
\begin{align*}
P^*_i(t) &= \begin{cases} 
p^*_i(x^t) & \text{if } \{x^t \neq x\} \text{ or } \{x^t = x \text{ and } p^t \neq \tilde{p}\} \text{ for some } 0 \leq \tau \leq t-1 \vphantom{\left(\frac{1}{2}\right)} \\
\tilde{p}_i & \text{otherwise}
\end{cases}
\end{align*}
$$

is a subgame perfect equilibrium pricing strategy provided that:

$$
\pi_i(x, \tilde{p}) - \delta \pi_i(x, p^*_b(x)) \geq (1 - \delta) \max_{p_i} \pi_i(x, p_i, \tilde{p}_-i).
$$

Equation (14) ensures that deviation from the prices $\tilde{p}$ is not profitable for any incumbent firm, while the initial entry process and the pricing strategy (13) guarantee that new entrants will not be attracted to the market. MacLeod et al. do not identify what the prices $\tilde{p}$ might be and it is to this question that we now turn.

4. Price Equilibrium in the Repeated Price Game

In determining the prices $\tilde{p}$ we need to identify first, the “shape” of the firms’ delivered price schedules and secondly, their one-period profit-maximizing deviation from these prices. We simplify the analysis by assuming that at the entry stage each firm is committed to a particular pricing policy. In the non-discriminatory case, this implies that all firms are committed to mill pricing whether or not they abide by the agreed prices.\(^7\) If the firms adopt discriminatory pricing, we assume that they are committed to a uniform delivered pricing policy.\(^8\)

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\(^7\) This implies that with mill pricing the consumers control collection of the goods. In the product differentiation analogy, it implies that the deviating firm is committed at the entry stage to a specialized technology that cannot produce a range of differentiated products.

\(^8\) This is not unreasonable given our assumption that demand is perfectly inelastic for all prices less than the reservation price $v$. In the production differentiation analog, the uniform pricing assumption implies that all firms are committed to charging the same price for all of the product variants they offer. It is interesting to note that Ford
It should be clear that the optimal one-period deviation from a tacitly agreed uniform delivered price is a uniform delivered price.

Assume for the moment that the market is of unit length\(^9\) and that it contains \(n\) symmetrically located firms, that no further entry is possible and that the consumer reservation price is sufficiently high or the number of firms sufficiently great that firms are in the competitive regions of their demand curves (Salop 1979). For the moment we can suppose the no-entry condition to be determined either exogenously by institutional constraints or endogenously as the result of the free-entry process we have detailed in section 3.

### 4.1 Mill Pricing

The free-entry Bertrand-Nash equilibrium mill prices are:

\[
m^b_i = \frac{1}{n}, \quad i = 1...n.
\]

After the entry stage has been completed we assume that the firms reach a tacit agreement on a set of mill prices \(\bar{m}_i^a = \bar{m}/n\), \(i = 1...n\), with instantaneous flow profits to each firm inclusive of fixed costs of:

\[
\pi^a_i = \frac{\bar{m}}{n^2} \quad i = 1...n.
\]

We wish to identify the highest mill prices \(\bar{m}_i^a(\delta)\) which can be an SPE for the discount factor \(\delta\). As we have noted, these prices must exactly balance the one-period rewards from cheating on the agreed prices against the long-term losses when such deviation is punished. The punishment phase is easily defined: we assume that firms revert to the one-shot Bertrand-Nash prices of equation (15) forever. If firm \(i\) decides to deviate from the tacitly agreed prices more complex considerations arise. In particular, firm \(i\) must decide how many of its neighboring firms it will undercut (stealing their entire markets).

It is convenient to use the following definition.

\[
(D) \quad \text{Let } h(\delta) = \frac{2\delta - 1 + \sqrt{3\delta^2 - 3\delta + 1}}{2(1 - \delta)} \quad \text{and define } j[\lfloor \delta \rfloor] \text{ to be the smallest integer } j \text{ greater than } h(\delta).
\]

Motor Company, for example, is increasingly employing this type of pricing policy over parts of the product spectrum that they feel to be reasonably distinct.

\(^9\) This normalization does not affect the analysis so long as the consumer reservation price is non-binding. We shall relax this assumption in section 4.2 when we consider the effects of a binding reservation price.
Note that \( j[\delta] \) is increasing in \( \delta \).

We can show (see the Mathematical Appendix) that if firm \( i \) deviates from the agreed prices it will do so in such a way that it undercuts exactly \( j[\delta] \) neighboring firms on each side. Firm \( i \) will not, however, set prices such that it undercuts \( j[\delta] \) neighbors and takes part of the markets of the \((j[\delta]) + 1\)th neighbors. We then have the following:

**Proposition 1:**

The highest mill price that can be an SPE for the repeated price game when the market contains \( n \) symmetrically located firms, no further entry is possible and the discount factor is \( \delta \) is:

\[
\tilde{m}_i^a(\delta) = \min \left[ v - \frac{1}{2n}, \frac{\delta(1 + j[\delta] + 2j[\delta]^2) - j[\delta]((1 + 2j[\delta])1/2)}{\delta(1 + 2j[\delta]) - 2j[\delta]} \right].
\]

where \( j[\delta] \) is defined by \((D)\) and is such that \( 1 < j[\delta] < n/2 \).

It is easy to show, as we would expect,\(^{11}\) that \( \frac{\partial \tilde{m}_i^a(\delta)}{\partial \delta} > 0 \). The upper limit on the SPE mill price is an increasing function of the discount factor, or equivalently, of the degree of price flexibility.

### 4.2 Discriminatory Pricing

We know\(^{12}\) that provided no firm has monopoly power in any segment of its market area the Bertrand-Nash equilibrium discriminatory pricing schedule for each entrant is:

\[
p_i^D(r|x) = \max \left( \|x_i - r\|, \min_j \|x_j - r\| \right), \quad i = 1...n.
\]

Now assume that after the entry stage has been completed the firms tacitly agree each to charge a uniform delivered price \( \tilde{u} \). As with discriminatory pricing, we wish to identify the highest uniform delivered prices \( \tilde{u}_i^a(\delta) \) that can be an SPE for the discount factor \( \delta \). Once again, these have to balance reward and punishment, where now equation \((18)\) gives the prices that will apply forever in the event of cheating. With discriminatory pricing it is easy to identify the best possible deviation for firm \( i \): price should be reduced marginally below the tacitly agreed uniform

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\(^{10}\) Proofs of all propositions are given in the Mathematical Appendix.

\(^{11}\) This is most easily done numerically since, for any given \( n \), the mill price is a function only of \( \delta \).
delivered price $\tilde{u}^*_i(\delta)$, giving the cheating firm a market area of $\tilde{u}^*_i(\delta)/s = \tilde{u}^*_i(\delta)$ with our normalization $s = 1$. We then have:

**Proposition 2:** The highest uniform delivered price that can be an SPE when the market contains $n$ symmetrically located firms, no further entry is possible and the discount factor is $\delta > 0.5$ is:

$$\tilde{u}^*_i(\delta) = \min \left[ v, \frac{1 + \sqrt{\delta(2\delta - 1)}}{2(1 - \delta)} \cdot \frac{1}{n} \right].$$

As with the mill price, $\frac{\partial \tilde{u}^*_i(\delta)}{\partial \delta} > 0$: the upper limit on the uniform delivered price is an increasing function of the discount factor (or the degree of price flexibility).

5. **The Welfare Effects of Deregulating Firms’ Pricing Policies**

The analysis of the previous section identifies upper limits on the prices that firms can charge but we know from the Folk Theorem that any prices between these upper limits and the Bertrand-Nash prices can be sustained as equilibrium prices for the relevant discount factor $\delta$. This makes welfare comparison of the alternative pricing policies impossible without the imposition of some further structure on the model. The approach we adopt is to assume that self-interest among the incumbent firms leads them to settle on the highest possible prices that can be charged consistent with there being no incentive to deviate. In other words, for the remainder of this paper, we assume that equations (17) and (19) describe the actual prices that will be charged by the incumbent firms after the entry stage has been completed.

A very simple result characterizes the comparison of discriminatory and mill prices for the one-shot price subgame, as illustrated by Figure 1. If there are $n$ active firms with each pricing policy as in Figure 1(a) no consumer loses from discriminatory pricing. By contrast, if there are $n$ active firms with mill pricing but only $n/2$ with discriminatory pricing as in Figure 1(b) no consumer gains from discriminatory pricing.

(Figure 1 near here)

It turns out that a similar property holds for the SPE prices of equations (17) and (19).

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12 See, for example, Gee (1976), Lederer and Hurter (1986).
**Proposition 3:** Assume that the reservation price is not binding and that there are \( \bar{n} \) active firms with mill pricing.

(i) \( \tilde{u}_i^n(\delta)|_{n=\bar{n}} < \tilde{m}_i^n(\delta)|_{n=\bar{n}} \quad \forall \ \bar{n} \geq 2 \): If there are also \( \bar{n} \) active firms with discriminatory pricing, all consumers gain from discriminatory pricing.

(ii) If no consumer is to gain from discriminatory pricing, there will have to be no more than \( \bar{n}/2 \) firms with discriminatory pricing.

These are actually stronger results than for the one-shot price subgame. It would appear that the advantage to consumers of discriminatory pricing relative to mill pricing is even greater with the repeated price game. This might on first sight seem surprising since, as Figure 1(a) makes clear, the punishment phase with discriminatory pricing is much more severe than with mill pricing. The reason is to be found in the very different rewards that accrue to price-undercutting with the two pricing policies. When firms employ mill pricing, a firm that wishes to cut price to one set of consumers must also offer reduced prices to all of its existing customers, dissipating at least some of the benefits of the reduced prices. A price-cutting firm suffers no such loss of revenue with discriminatory pricing since prices to its existing customers are maintained effectively unchanged, strengthening the temptation to cut prices. The need to offset the one-period gains to price-cutting imposes a sufficiently strong constraint on the tacitly agreed discriminatory prices as to lead to Proposition 3.

In order to extend our comparisons of the two pricing policies we need to be more precise about the outcome of the entry stage detailed in section 3. A key property that affects entry, market structure and so the effects of price deregulation is the degree of spatial contestability. When incumbents can relocate their activities costlessly the industry is defined to be spatially contestable (SC). At the other extreme, if relocation costs are prohibitive, making location choice once-for-all, the industry is defined to be spatially non-contestable (SNC).

In a free-entry equilibrium with SC each entrant firm just breaks even. We take as the free-entry SNC equilibrium the loosest packing of firms consistent with there being no further
entry. At this equilibrium the incumbents fully exploit the entry-deterring advantage of being committed to their locations. As a result, the free entry number of entrants is:

\[
\begin{align*}
    n^*_d &= (2f)^{-1/2} : n^*_f &= (f)^{-1/2} \\
    n^{SNC}_d &= (8f)^{-1/2} : n^{SNC}_f &= \frac{\sqrt{3/2}}{\sqrt{2 + \sqrt{3}}f}.
\end{align*}
\] (20)

Since \( n^*_d / n^*_f > 1/2 \) we cannot conclude that the entry-deterring effect of discriminatory pricing more than offsets its pro-competitive benefits for consumers. We must, therefore, turn to more aggregate comparisons. In doing so, it turns out that we must also distinguish between cases in which the reservation price is not binding and those in which it is.

5.1 Welfare Comparisons: Non-Binding Consumer Reservation Price

Throughout this subsection we shall assume that the consumer reservation price imposes no effective constraint on firms’ prices. The complex nature of the price equilibria makes analytical results difficult to derive. However, given equations (17), (19) and (20), set-up costs appear as a common multiplier provided that the consumer reservation price is not binding. As a result, all comparisons are determined by the discount factor \( \delta \) and numerical comparisons provide a complete description of the relative merits of the two pricing policies. Recall that \( \delta = e^{-\gamma \rho} \), with the result that, for a given continuous rate of time discount, \( \delta \) will be greater the faster the speed of response of incumbents (the lower is \( \gamma \)). Figure 2 and Table 1 summarize the welfare effects of the two pricing policies. In Table 1 we also repeat our results for the one-shot price subgame to facilitate comparison.

Consider, first, the impact of pricing policy on delivered prices (Figure 2(a)). If the market is SC every consumer is better off with discriminatory pricing for any discount factor greater than 0.5 and if the market is SNC every consumer is better off with discriminatory pricing provided the discount factor is greater than 0.601. The benefits to consumers of the pro-competitive effects of discriminatory prices with a given market structure extend to endogenous market structures when prices are determined by equations (17) and (19). Discriminatory pricing increases market concentration and so increases prices (which is why the difference between the

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13 See Norman and Thisse (1996) for a detailed discussion of these results. The subscript \( d \) denotes discriminatory pricing and the subscript \( f \) denotes mill (or fob) pricing.
mill price and uniform delivered price is lower when the market is SNC). However, this is more than offset by the need to make price-undercutting unprofitable which, as we have already indicated, imposes a stronger constraint on pricing when firms employ discriminatory pricing.

The impact of discriminatory pricing on aggregate consumer surplus is straightforward. If firms’ total revenue is denoted \( TR(n) \) and aggregate consumer surplus is denoted \( CS(n) \) then it follows from our definition of individual consumer demand that \( CS(n) = v - TR(n) \). From Figure 2(b) mill pricing generates higher total revenues and so lower aggregate consumer surplus than discriminatory pricing no matter the degree of spatial contestability. This is in contrast to the one-shot price subgame, in which mill pricing benefits consumers if the market is SNC. When the price subgame is repeated, the ability of firms to raise prices when they employ a mill pricing policy is sufficiently strengthened that, no matter the degree of spatial contestability, consumers unambiguously benefit from discriminatory prices.

Now consider the impact of pricing policy on profitability (Figure 2(c)). If the market is SC (SNC) mill (discriminatory) pricing gives greater supernormal profits to the individual firm.\(^{14}\) Individual firm profitability with the two pricing policies is determined by the interplay between the price-reducing and entry-deterring effects of allowing firms to employ a discriminatory pricing policy. If the market is SC the price-reducing effect is dominant. By contrast, if the market is SNC equations (20) indicate that the entry-deterring effect of price deregulation is strengthened. This is sufficient, at the individual firm level, to offset the profit-reducing effects of fiercer price competition.

(Figure 2 and Table 1 near here)

Such ambiguity does not characterize aggregate profits: Figure 2(d). Aggregate profit is greater with mill pricing than discriminatory pricing no matter the degree of spatial contestability. This is in sharp contrast to our analysis of SNC with a one-shot price subgame: see Table 1. The difference derives once again from our discussion of Proposition 3 in which we saw that the competitive disadvantage of discriminatory pricing is strengthened when the price subgame is repeated.

\(^{14}\) There is a slight exception to this. With SC the difference in profits between mill and discriminatory pricing is a series of quadratics, one for each value of \( j \). Each such quadratic has a turning point at the discount factor \( \delta(j) = 4j^2/(4j^2 + 4j - 1) \) at which individual profits are equal for the two types of firm.
The comparison of total surplus is just as in Norman and Thisse (1996). If total costs are $TC(n)$ then total surplus is $v - TC(n)$. Since the entry process we have assumed is independent of the nature of the post-entry price subgame, total costs are also independent of the nature of the post-entry price subgame. If the market is SNC (SC) mill pricing will generate higher (lower) total surplus than discriminatory pricing.

To summarize, when the price subgame is repeated and the consumer reservation price is non-binding, discriminatory prices benefit consumers almost always individually and always in the aggregate while mill prices benefit producers in the aggregate (and individually if the market is SC). Although the impact of discriminatory prices on aggregate surplus is determined by the degree of spatial contestability when consumer surplus and producer surplus are weighted equally, it is clear that there will be some additional weighting of consumer surplus above which regulators should unambiguously favor discriminatory prices.

It is important to note, however, that our results in this subsection are based upon the assumption that the consumer reservation price imposes no effective limit on firms’ choice of prices as given by equations (17) and (19). We now examine the effect of relaxing this assumption.

5.2 Welfare Comparisons: Binding Consumer Reservation Price

It will emerge below that if the consumer reservation price is binding, welfare comparisons of the two pricing policies will be affected by market length. As a result, assume that the market is of length $\Lambda$. Given the entry process described in section 3, the free-entry number of entrants is now equal to:

\[
\begin{align*}
n^S_{d} &= \left(\Lambda/2f\right)^{1/2}; n^S_{f} = \left(\Lambda/f\right)^{1/2} \\
n^S_{SC} &= \left(\Lambda/8f\right)^{1/2}; n^S_{f} = \frac{\sqrt{3}\Lambda/2}{\sqrt{(2 + \sqrt{3})}f}.
\end{align*}
\]

We know that the highest mill price any firm will wish to charge is $v - \Lambda/2n$ and that the highest uniform delivered price that can be charged is $v$. Also, an important implication of subsection 5.1 is that, no matter the degree of spatial contestability, the discount factor $\delta_f$ above which the consumer reservation price is binding is lower with mill pricing than with discriminatory pricing. We take advantage of these two properties in this section. Our discussion
turns the analysis of the previous section on its head to an extent by seeking to answer the following questions:

(i) Under what circumstances will all consumers be worse off with discriminatory pricing as compared to mill pricing and how is this affected by the degree of spatial contestability?

For this to be the case it must be that the uniform delivered price is exactly equal to the consumer reservation price \( v \), since we know that the maximum delivered price with mill pricing is \( v \) and this applies to a zero measure of consumers. So what does the critical discount factor \( \hat{\delta}_d \) have to be for a uniform delivered price \( \tilde{u}_i(\hat{\delta}_d) = v \) to be sustainable as an SPE? We can then show how this critical discount factor is affected by market size, the consumer reservation price and the firms’ set-up costs.

(ii) Under what circumstances will consumers on average be worse off with discriminatory pricing and how is this affected by the degree of spatial contestability?

For this to be the case the uniform delivered price must be greater than \( v - \Lambda/4n \). So above what discount factor will a uniform delivered price of \( v - \Lambda/4n \) be sustainable as an SPE? Again, we can investigate how this discount factor is affected by market size, the consumer reservation price and the firms’ set-up costs.

(iii) Under what circumstances will firms either individually or on aggregate benefit from price deregulation and how is this affected by the degree of spatial contestability?

The approach we adopt is straightforward. Equation (19) identifies the maximum uniform delivered price that is sustainable as an SPE for a given discount factor. Inverting this equation, therefore, gives us the lowest discount factor for which a particular uniform delivered price is sustainable as an SPE. Assume that the uniform delivered price is \( \kappa/n \leq v \) for some parameter \( \kappa \geq 1 \). From equation (19), the discount factor \( \hat{\delta}_d(\kappa) \) for which the uniform delivered price \( \tilde{u}_i(\hat{\delta}_d(\kappa)) \) equals \( \kappa/n \) is:

\[
(21) \quad \hat{\delta}_d(\kappa) = \frac{(2\kappa - 1)^2}{2(2\kappa^2 - 1)} \geq 0.5 \text{ for } \kappa \geq 1.
\]

It follows immediately from (21) that \( \partial \hat{\delta}_d(\kappa)/\partial \kappa \geq 0 \). In other words, the critical discount factor \( \hat{\delta}_d(\kappa) \) is increasing in \( \kappa \).
Consider first the critical discount factor above which *all* consumers will be worse off with discriminatory pricing. We have already noted that for this to be the case 
\[ \tilde{u}_i^a(\bar{d}_i(\kappa_1)) = \kappa_1/n = v \]
so that \( \kappa_1 = v.n \), and from equation (20a) we have:

\[
(22) \quad \kappa_1^{SC} = \frac{\sqrt{\Lambda}}{2} > \kappa_1^{SNC} = \frac{\hat{v}}{2} \sqrt{\frac{\Lambda}{2}}
\]

where \( \hat{v} = v/\sqrt{f} \). When the consumer reservation price imposes an effective constraint on firms’ mill prices, the critical discount factor above which all consumers are worse off with discriminatory pricing is lower when the market is SNC than when it is SC.

For consumers in the aggregate to lose from discriminatory pricing it must be the case that 
\[ \tilde{u}_i^a(\bar{d}_i(\kappa)) \geq \kappa_2/n = v - \Lambda/4n \], which gives a lower limit on \( \kappa \) of \( \kappa_2 = v.n - \Lambda/4 \).

Substituting from equation (20a) gives:

\[
(23) \quad \kappa_2^{SC} = \frac{\hat{v}}{\sqrt{2}} - \frac{\Lambda}{4} > \kappa_2^{SNC} = \frac{\hat{v}}{2} \sqrt{\frac{\Lambda}{2}} - \frac{\Lambda}{4}
\]

and we obtain the same qualitative conclusion.

Note further that \( \partial \kappa_i^*/\partial \hat{v} > 0 \) and \( \partial \kappa_i^*/\partial \Lambda > 0 \) \( (i = 1,2) \) implying that when mill prices are constrained by the reservation price, consumers individually and in the aggregate are more likely to lose from discriminatory pricing if the market is “small”, the reservation price is “low” and/or firms’ set-up costs are “high”.

For firms individually to benefit from discriminatory pricing the lower limit on \( \kappa \) is such that (see the Mathematical Appendix):

\[
(24) \quad \begin{cases} \kappa_3^{SC} = 0.5\hat{v}\sqrt{\Lambda} \\ \kappa_3^{SNC} = 0.079\hat{v}\sqrt{\Lambda} + 0.225 \end{cases} \quad \Rightarrow \kappa_3^{SC} > \kappa_3^{SNC}.
\]

When mill prices are constrained by the reservation price, the critical discount factor above which firms individually gain from discriminatory pricing is lower when the market is SNC than when it is SC. By the same argument as in footnote 14, we have \( \partial \kappa_3^*/\partial \hat{v} > 0 \) and \( \partial \kappa_3^*/\partial \Lambda > 0 \), implying that when mill prices are constrained by the reservation price, firms individually are

\[ \text{It must be the case that } v \geq \Lambda/n \text{ and we know from (20a) that } n = \sqrt{\Lambda/\eta f} \text{ where } \eta \geq 1. \text{ It follows that } v/\sqrt{f} > \sqrt{\Lambda}. \]
more likely to gain from discriminatory pricing if the market is “small”, the reservation price is “low” and/or firms’ set-up costs are “high”.

Finally, we can show that for firms in the aggregate to gain from discriminatory pricing the lower limit on $\kappa$ is:

$$
\begin{align*}
\kappa_{4}^{SC} &= 0.707\sqrt{\Lambda} - 0.104\Lambda - 0.207 \\
\kappa_{4}^{SNC} &= 0.354\sqrt{\Lambda} + 0.138\Lambda - 0.433
\end{align*}
\Rightarrow \kappa_{4}^{SC} > \kappa_{4}^{SNC}.
$$

When mill prices are constrained by the reservation price, the critical discount factor above which firms will gain from discriminatory pricing is lower when the market is SNC than when it is SC. Once again, we have $\partial \kappa_{4}^{*} / \partial \hat{\theta} > 0$ and $\partial \kappa_{4}^{*} / \partial \Lambda > 0$, implying that when mill prices are constrained by the reservation price, firms in the aggregate are more likely to gain from discriminatory pricing if the market is “small”, the reservation price is “low” and/or firms’ set-up costs are “high”.

These results take us back more nearly to those presented in Norman and Thisse for the one-shot price subgame. When mill prices are constrained by the consumer reservation price, there will be a degree of price flexibility above which consumers will lose and producers will benefit from discriminatory pricing. The degree of price flexibility necessary for this to be the case is lower if the market is SNC than if it is SC, reflecting the stronger entry-deterring nature of SNC.

We are also brought back to our question in the introduction of whether there is a potential role for the regulatory authorities in influencing the degree of price flexibility. Equations (17) and (19) indicate that, no matter the allowable pricing policy, prices can be expected to be higher the more flexible are firms’ pricing decisions: since a high degree of price flexibility translates to a high discount factor. We have shown that when the consumer reservation price does not impose an effective constraint on firms, consumers always benefit from price deregulation. We now see that these benefits will be even greater if price flexibility can be reduced. When the consumer reservation price does impose an effective constraint, our analysis indicates that consumers will not gain from price deregulation, particularly in more concentrated markets, unless the degree of price flexibility is sufficiently low, where the definition of "sufficient" can be implied from our formal analysis.
It would appear, therefore, that there are, indeed, consumer benefits to be had if price deregulation is accompanied by policies that reduce price flexibility, perhaps by the regulator imposing some type of minimum period within which prices cannot be changed. The intuition behind this is familiar from the Folk Theorem. Low price flexibility reduces the effectiveness of the punishment phase that sustains the tacitly collusive prices above the one-shot Bertrand-Nash level and so lowers the maximum sustainable prices.

6. Conclusions

In this paper we have extended our earlier analysis of the relative merits of soft and tough price regimes to the situation in which post-entry price equilibria reflect the repeated nature of the post-entry price subgame. We derive welfare conclusions that contrast reasonably sharply with our one-shot analysis. If the constraint implicit in the consumer reservation price is non-binding then aggregate consumer surplus is greater and aggregate profits are lower with discriminatory pricing than with mill pricing no matter the degree of spatial contestability of the market. In addition, provided that price decisions are sufficiently flexible ($\delta > 0.601$) every consumer will benefit from price discrimination. The entry-deterring effect of discriminatory pricing is more than offset by its pro-competitive effect.

The reason for this contrast lies in the very different constraints the two pricing policies impose on firms’ abilities to raise their prices above the Bertrand-Nash prices. Such constraints exist since any set of tacitly agreed prices must be sensitive to the temptation each firm has to cheat on these prices and so must be supported by some credible punishment strategy in the event that cheating occurs. The punishment strategy on which we have focused in this paper is the grim trigger strategy in which, if deviation occurs, firms move to the one-shot Bertrand-Nash prices forever. Given the relatively aggressive entry process we have proposed, we know from our previous analysis that at these prices firms individually and in the aggregate earn at least as great profits with discriminatory pricing as with mill pricing. With a mill pricing policy any attempt by one firm to undercut its rivals requires that the firm also offers the lower prices to all of its existing customers. There is no such requirement with discriminatory pricing as a result of which the gains from cheating are greater with discriminatory pricing than with mill pricing. This combination of weaker punishment of and greater gains to cheating with a discriminatory pricing policy leads to the consumer gains and producer losses we have noted.
If our analysis were to stop here it would appear that we could be much more sanguine about the benefits of deregulation of firms’ pricing policies even if this were to affect market structure in the ways we have suggested. Our analysis has also shown, however, that the consumer reservation price has an important part to play in determining the welfare properties of alternative pricing policies. If the reservation price effectively constrains mill prices then there exists a degree of price flexibility above which our welfare conclusions are reversed, with firms gaining and consumers losing from discriminatory pricing. The degree of price flexibility above which this will apply is greater when the market is SNC; when market size is small; when the consumer reservation price is low; and when firms’ set-up costs are high.

Each of these conditions implies that there are less likely to be benefits to consumers from price deregulation in markets already characterized by high levels of market concentration. They further imply a role for the regulatory authorities that has, so far as we are aware, not been considered in the literature, primarily because most of the pre-existing analysis is static rather than dynamic. We have shown that the discount factor has a crucial role to play in determining the benefits that consumers are likely to derive from price deregulation. Simply put, consumers are likely to gain more from price deregulation if the discount factor is low. The discount factor is itself determined by the (continuous) rate of time discount, which is generally outside the control of the regulatory authorities, and the speed with which firms can react to deviation from the tacit agreed prices, which is conceivably within the control of the authorities. Our analysis indicates that price deregulation is more likely to be successful if it is accompanied by policies that impose a period over which prices cannot be changed. Interestingly, this is especially likely to be the case in concentrated markets and these are the markets in which response speeds might be expected to be greatest.
Table 1: Welfare Comparisons

<table>
<thead>
<tr>
<th></th>
<th>Spatial Contestability</th>
<th>Spatial Non-Contestability</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Repeated Price Game</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prices</td>
<td>$\tilde{m}_i^a(\delta) &gt; \tilde{u}_i^a(\delta)$ if $\delta &gt; 0.5$</td>
<td>$\tilde{m}_i^a(\delta) &gt; \tilde{u}_i^a(\delta)$ if $\delta &gt; 0.601$</td>
</tr>
<tr>
<td>Consumer Surplus</td>
<td>$CS_f &gt; CS_d$</td>
<td>$CS_f &gt; CS_d$</td>
</tr>
<tr>
<td>Supernormal Profit</td>
<td>$\pi_{if} &gt; \pi_{id}$</td>
<td>$\pi_{if} &lt; \pi_{id}$</td>
</tr>
<tr>
<td></td>
<td>$\Pi_f &gt; \Pi_d$</td>
<td>$\Pi_f &gt; \Pi_d$</td>
</tr>
<tr>
<td>Total Surplus</td>
<td>$TS_f &lt; TS_d$</td>
<td>$TS_f &gt; TS_d$</td>
</tr>
<tr>
<td><strong>One-Shot Price Game</strong></td>
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<tr>
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<td>$CS_f &gt; CS_d$</td>
</tr>
<tr>
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<td>Total Surplus</td>
<td>$TS_f &lt; TS_d$</td>
<td>$TS_f &gt; TS_d$</td>
</tr>
</tbody>
</table>

Notes: $^a$ The welfare comparisons relate to cases in which the consumer reservation price is non-binding.
(a) $n$ Firms with Each Pricing Policy

(b) $n$ Firms with Mill Pricing; $n/2$ Firms with Discriminatory Pricing

Figure 1: Comparison of Price Equilibria
Price Regulation and Tacit Collusion

Spatial Contestability

Spatial Non-Contestability

(a) Difference between Mill Price and Uniform Delivered Price

(b) Difference in Total Revenue

(c) Difference in Individual Firm Profits

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(d) *Difference in Aggregate Profit*

*Figure 2: Welfare Comparison of Non-Discriminatory versus Discriminatory Pricing*
Mathematical Appendix

Proof of Proposition 1:

(a) Assume that firms have tacitly agreed on the mill price \( m/n \), where \( m \geq 1 \). Assume further that a price-cutting firm charges a price \((m - j)/n\) that just undercuts \( j \geq 1 \) neighboring firms. This gives a market radius to the price-cutting firm of \( j/n + 1/2n \) and instantaneous gross profit of:

\[
\pi^c = (m - j)(2j + 1)/n^2.
\]

Instantaneous gross profit with the agreed mill price \( m/n \) is:

\[
\pi^a = m/n^2.
\]

The Bertrand-Nash equilibrium mill price is \( 1/n \) with instantaneous gross profit

\[
\pi^b = 1/n^2.
\]

For such price-cutting to be unprofitable we require that:

\[
\pi^a - \delta \pi^b \geq (1 - \delta)\pi^c.
\]

Solving for \( m \) gives the upper bound on the tacitly agreed price:

\[
\frac{\tilde{m}(\delta, j)}{n} = \frac{\delta(1 + j + 2j^2) - j(1 + 2j)}{\delta(1 + 2j) - 2j} \cdot \frac{1}{n}.
\]

By a similar analysis, if the price-cutting firm just undercuts \( j + 1 \) neighbors the upper bound on the mill price is:

\[
\frac{\tilde{m}(\delta, j + 1)}{n} = \frac{\delta(4 + 5j + 2j^2) - (j + 1)(3 + 2j)}{\delta(3 + 2j) - 2(j + 1)} \cdot \frac{1}{n}.
\]

These upper bounds are equal when \( \delta(j) = \frac{4j(j + 1)}{1 + 8j + 4j^2} \). Thus the upper bound on the mill price is \( \frac{\tilde{m}(\delta, j)}{n} \) for \( \delta \in [\delta(j - 1), \delta(j)] \) for the price-cutting firm to wish just to undercut exactly \( j \) neighbors to each side. Inverting \( \delta(j) \) gives \( h(\delta) \) in the text.

(b) We now check that the price-cutting firm would not prefer an interior solution in which it undercuts \( j \) neighbors to each side and takes only part of the market of the \((j+1)\)th neighbors.

If the tacitly agreed mill price is \( \frac{\tilde{m}(\delta, j)}{n} \) and the price-cutting firm charges a price \( c/n \) it gets market radius \( (1 + \tilde{m}(\delta, j) + j - c)/2n \) and earns instantaneous gross profit of \( c(1 + \tilde{m}(\delta, j) + j - c)/n \) giving the optimal price:

\[
\frac{c(\delta, j)}{n} = \frac{1 + \tilde{m}(\delta, j) + j}{2n}.
\]

We need merely check that \( (c(\delta, j) + j)/n \geq m(\delta, j)/n \). Substituting from (A.5) and simplifying gives:
(A.7) \[ \Delta = c(\delta, j) + j - \tilde{m}(\delta, j) = \frac{j(4\delta(j+1)-(4j+1))}{2(\delta(2j+1)-2j)}. \]

Differentiating \( \Delta \) with respect to \( \delta \) gives:

(A.8) \[ \frac{\partial \Delta}{\partial \delta} = \frac{(1-2j)j}{2(\delta(2j+1)-2j)^2} < 0 \text{ for } j \geq 1. \]

Thus we need only evaluate \( \Delta \) at \( \delta(j) \). This gives:

(A.9) \[ \Delta|_{\delta=\delta(j)} = \frac{2j-1}{4} > 0. \]

It follows that the corner solution (A.5) is always the preferred method for a price-cutting firm.

**Proof of Proposition 2:**

Assume that firms have tacitly agreed on a uniform delivered price \( u \), where \( u \geq 1/n \). A price-cutting firm will just undercut this price, giving it a market radius of \( u/n \) and instantaneous gross profit:

(A.10) \[ \pi^c = 2\int_{r=0}^{u/n} \left( \frac{u}{n} - r \right) dr = u^2/n^2. \]

Instantaneous gross profit with the agreed uniform delivered price is:

(A.11) \[ \pi^a = 2\int_{r=0}^{4u-1/4} \left( \frac{u}{n} - r \right) dr = (4u-1)/4n^2. \]

The Bertrand-Nash equilibrium prices give instantaneous gross profits of:

(A.12) \[ \pi^b = 1/2n^2. \]

For price-cutting to be unprofitable we require that:

\[ \pi^a - \delta \pi^c \geq (1-\delta)\pi^c. \]

Solving for \( u/n \) gives the upper bound on the tacitly agreed uniform delivered price:

(A.13) \[ \tilde{u}(\delta) = \frac{1+\sqrt{\delta(2\delta-1)}}{2(1-\delta)} \cdot \frac{1}{n}. \]

The upper limit on this price is, of course, the consumer reservation price \( v \).

**Proof of Proposition 3:**

(i) For a given \( n \) and provided that the reservation price is not binding, the difference between the tacitly agreed mill and uniform delivered prices is:

(A.14) \[ \Delta_1 = \frac{\delta(1+j[\delta]+2j[\delta]^2) - j[\delta]^2((1+2j[\delta])} {\delta(1+2j[\delta]) - 2j[\delta]} \cdot \frac{1}{n} \cdot \frac{1+\sqrt{\delta(2\delta-1)}}{2(1-\delta)} \cdot \frac{1}{n} \]

For a given \( n \) this is a function solely of the discount factor \( \delta \) and so can be examined numerically. This examination confirms that \( \Delta_1 > 0 \) as required.
(ii) For a given \( n \) and provided that the reservation price is not binding, the difference between the maximum delivered mill price with \( n \) firms and the uniform delivered price with \( n/2 \) firms is:

\[
\Delta_2 = \frac{\delta(1 + j[\delta][1 + 2j[\delta]])}{\delta(1 + 2j[\delta]) - 2j[\delta]} \cdot \frac{1 + \frac{1 + \sqrt{\delta(2\delta - 1)}}{2(1 - \delta)}}{n}.
\]

For a given \( n \) this is a function solely of the discount factor \( \delta \) and so can be examined numerically. This confirms that \( \Delta_2 > 0 \) but that \( \Delta_2 = 0 \) for exactly one value of \( \delta \) for each value of \( j[\delta] \). It follows that for at least some consumers to prefer mill pricing there will have to be fewer than \( n/2 \) firms with uniform delivered pricing.

Proof of Equations (24) and (25):

If the mill price is constrained to \( v - \Lambda/2n \) then individual firm and aggregate profit are respectively:

\[
\begin{align*}
\pi_f &= \Lambda v/n_f - f - \Lambda^2/2n_f^2 \\
\Pi_f &= \Lambda v - f \cdot n_f - \Lambda^2/2n_f
\end{align*}
\]

(A.16)

where from equation (20a) we know that:

\[
(\Lambda \alpha)^2 = \frac{1 + \sqrt{\delta(2\delta - 1)}}{2(1 - \delta)}
\]

(A.17)

with \( \alpha = 1 \) or \( 3/(2(2+\sqrt{3})) \). Substituting in (A.16) gives:

\[
\begin{align*}
\pi_f &= \frac{f}{2} \left( \hat{\alpha} - \frac{\Lambda}{\alpha} - 2 \right) \\
\Pi_f &= \frac{\Lambda}{\alpha} \left( \hat{\alpha} - \frac{\Lambda}{\alpha} - 2 \right)
\end{align*}
\]

(A.18)

Assume that the uniform delivered price is \( u/n_d \). Then individual firm and aggregate profit with uniform delivered pricing are:

\[
\begin{align*}
\pi_d &= \Lambda u/n_d^2 - f - \Lambda^2/4n_d^2 \\
\Pi_d &= \Lambda u - f \cdot n_d - \Lambda^2/4n_d
\end{align*}
\]

(A.19)

and we know that:

\[
(\Lambda \beta)^2 = \frac{1 + \sqrt{\delta(2\delta - 1)}}{2(1 - \delta)}
\]

(A.20)

where \( \beta = 2 \) or 8. Substituting in (A.19) gives:

\[
\begin{align*}
\pi_d &= \frac{f}{4} \left( 4\beta u - \beta \Lambda - 4 \right) \\
\Pi_d &= \frac{\Lambda}{\beta} \left( 4\beta u - \beta \Lambda - 4 \right).
\end{align*}
\]

(A.21)

Solving \( \pi_d \geq \pi_f \) and \( \Pi_d \geq \Pi_f \) for \( u \) gives the critical values of \( \kappa \) above which \( \pi_d \geq \pi_f \) and \( \Pi_d \geq \Pi_f \). Substituting for \( \alpha \) and \( \beta \) in these equations gives equations (24) and (25).
REFERENCES